Vector Dark Matter from a Dark SU(2) Gauge Theory

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https://yzhxxzxy.github.io

Based on Zexi Hu, Chengfeng Cai, Yi-Lei Tang, Zhao-Huan Yu, Hong-Hao Zhang, arXiv:2103.00220, JHEP

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Vector Dark Matter

🌺 **Vector dark matter** (DM) consists of **spin-1** vector bosons

⭐ If **extra dimensions** exist, the first Kaluza-Klein mode of the $\mathrm{U}(1)_\mathrm{Y}$ gauge boson could be a well motivated vector DM candidate

[Servant & Tait, hep-ph/0206071, NPB; HC Cheng, JL Feng & Matchev, hep-ph/0207125, PRL]

🌙 **Gauge theories** in the 4D spacetime 👉 renormalizable vector DM models

🌘 Stueckelberg/Brout-Englert-Higgs mechanism 👉 gauge boson **mass**

🌗 At least one **gauge boson** acts as the **DM particle**

🌖 For a dark $\mathrm{U}(1)$ gauge field A^μ, a Z_2 symmetry $A^\mu \to -A^\mu$ must be imposed to forbid the kinetic mixing with the $\mathrm{U}(1)_\mathrm{Y}$ gauge field that leads to DM decays

[Lebedev, HM Lee & Mambrini, 1111.4482, PLB; ...]

🌕 **Non-abelian dark gauge groups**: 🍅 $\mathrm{SU}(2)$ [Hambye, 0811.0172, JHEP; ...]

- 🍒 $\mathrm{SU}(2)\times\mathrm{U}(1)$ [CW Chiang, Nomura & Tandean, 0811.0172, JHEP; ...]
- 🍇 $\mathrm{SU}(2)\times\mathrm{SU}(2)$ [Abe, Fujiwara, Hisano & Matsushita, 0811.0172, JHEP]
- 🍉 $\mathrm{SU}(3)$ and general $\mathrm{SU}(N)$ [Gross, Lebedev & Mambrini, 1505.07480, JHEP; Di Chiara & Tuominen, 1506.03285, JHEP; ...]

Non-abelian Dark Gauge Symmetries

- A **dark** $\mathrm{SU}(2)$ gauge symmetry
  - Spontaneously broken by **one** $\mathrm{SU}(2)$ **Higgs doublet**
    - **Three degenerate** gauge bosons acting as vector DM particles
    - A remaining **custodial global** $\mathrm{SU}(2)$ **symmetry** ensures the stability of vector DM [Hambye, 0811.0172, JHEP]
  - Spontaneously broken by **one real** $\mathrm{SU}(2)$ **Higgs triplet**
    - **Two degenerate** gauge bosons acting as vector DM particles
    - A $\mathrm{U}(1)$ **gauge symmetry** remains, leading to a **massless** gauge boson serving as **dark radiation** [S Baek, P Ko & WI Park, 1311.1035, JHEP]
  - Spontaneously broken by **two real** $\mathrm{SU}(2)$ **Higgs triplets** (this work)
    - Three gauge bosons can obtain **totally different masses**
    - Two lighter gauge bosons are odd under a remaining Z_2 **symmetry**, and the lightest one is **stable**, acting as a vector DM particle
- For a general **dark** $\mathrm{SU}(N)$ gauge group, all the gauge bosons can be made massive if $N-1$ **Higgs fields** in the **fundamental** representation are introduced

Our Model

- We consider a **dark $\mathrm{SU}(2)_\mathrm{D}$ gauge symmetry** broken by **two real Higgs triplets**, Φ_a and X_a $(a = 1, 2, 3)$, leading to massive $\mathrm{SU}(2)_\mathrm{D}$ **gauge fields** $\tilde{A}^a_\mu$

- $$\mathcal{L} \supset -\frac{1}{4}\tilde{A}^a_{\mu\nu}\tilde{A}^{a,\mu\nu} + \frac{1}{2}(D_\mu\Phi_a)^\mathrm{T}(D^\mu\Phi_a) + \frac{1}{2}(D_\mu X_a)^\mathrm{T}(D^\mu X_a) - V_\mathrm{SM} - V_\mathrm{D} - V_\mathrm{P}$$

$$\tilde{A}^a_{\mu\nu} = \partial_\mu\tilde{A}^a_\nu - \partial_\nu\tilde{A}^a_\mu + g_\mathrm{D}\varepsilon^{abc}\tilde{A}^{b,\mu}\tilde{A}^{c,\nu},\quad D_\mu\Phi_a = \partial_\mu\Phi_a + g_\mathrm{D}\varepsilon^{acb}\tilde{A}^c_\mu\Phi_b,\quad D_\mu X_a = \partial_\mu X_a + g_\mathrm{D}\varepsilon^{acb}\tilde{A}^c_\mu X_b$$

- SM potential $V_\mathrm{SM} = -\mu_0^2|H|^2 + \lambda_0|H|^4$ with the $\mathrm{SU}(2)_\mathrm{L}$ Higgs doublet H

- Dark potential $V_\mathrm{D} = -\mu_1^2\Phi_a\Phi_a - \mu_2^2 X_a X_a - \mu_3^2\Phi_a X_a + \lambda_1(\Phi_a\Phi_a)^2 + \lambda_2(X_a X_a)^2$
$$+\lambda_3\Phi_a\Phi_a X_b X_b + \lambda_4\Phi_a\Phi_a\Phi_b X_b + \lambda_5\Phi_a X_a X_b X_b + \lambda_6(\Phi_a X_a)^2$$

- Portal potential $V_\mathrm{P} = \lambda_{10}|H|^2\Phi_a\Phi_a + \lambda_{20}|H|^2 X_a X_a + \lambda_{30}|H|^2\Phi_a X_a$

- The Lagrangian respects an **accidental Z_2 symmetry**

$$\Phi \to P_\mathrm{D}\Phi = -\Phi,\quad X \to P_\mathrm{D}X = -X \text{ with } \textbf{dark parity } P_\mathrm{D} = \mathrm{diag}(-1,-1,-1)$$

- Z_2 + global $\mathrm{SO}(3)_\mathrm{D}$ = **global $\mathrm{O}(3)_\mathrm{D}$ symmetry**
  - $\mathrm{O}(3)_\mathrm{D}$ **vectors** $\Phi_a \to R_{ab}\Phi_b,\quad X_a \to R_{ab}X_b,\quad \forall R \in \mathrm{O}(3)_\mathrm{D}$
  - $\mathrm{O}(3)_\mathrm{D}$ **axial vector** $\tilde{A}^a_\mu \to \det(R)R_{ab}\tilde{A}^b_\mu$ 👉 $\tilde{A}^a_\mu$ is **P_D-even**

Spontaneous Symmetry Breaking

🌽 Generally, the vacuum expectation values (VEVs) $\langle\Phi_a\rangle$ and $\langle X_a\rangle$ are not parallel to each other, so they determine **a plane** in the 3D representation space

💡 We can always rotate the axes to a configuration that the z-axis is along the $\langle\Phi_a\rangle$ direction and the y-axis lies inside **the plane**

🎉 Without loss of generality, the Higgs fields can be expanded as

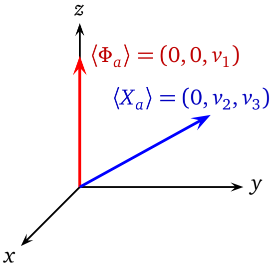
$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_0 + \tilde{h}_0 \end{pmatrix}, \quad \Phi = \begin{pmatrix} \phi_1 \\ \phi_2 \\ v_1 + \phi_3 \end{pmatrix}, \quad X = \begin{pmatrix} \chi_1 \\ v_2 + \chi_2 \\ v_3 + \chi_3 \end{pmatrix}$$

✨ The VEV configuration is preserved under the **reflection** with respect to the y-z **plane**

$$P'_{\mathrm{D}} = \mathrm{diag}(-1, +1, +1) \in \mathrm{O}(3)_{\mathrm{D}}$$

👉 A Z'_2 **symmetry** remains after the spontaneous breaking of the global $\mathrm{O}(3)_{\mathrm{D}}$ symmetry

- 🍎 P'_{D}**-odd fields** $\phi_1,\ \chi_1,\ \tilde{A}^2_\mu,\ \tilde{A}^3_\mu$
- 🍏 All the other fields are P'_{D}**-even**



Dark Gauge Bosons

🐅 The mass-squared matrix for the **dark gauge bosons** ($\tilde{A}^1_\mu$, $\tilde{A}^2_\mu$, $\tilde{A}^3_\mu$) is

$$\mathcal{M}_A^2 = g_{\rm D}^2 \begin{pmatrix} v_{123}^2 & & \\ & v_{13}^2 & -v_2 v_3 \\ & -v_2 v_3 & v_2^2 \end{pmatrix}, \quad v_{13} \equiv \sqrt{v_1^2 + v_3^2}, \quad v_{23} \equiv \sqrt{v_2^2 + v_3^2}, \quad v_{123} \equiv \sqrt{v_1^2 + v_2^2 + v_3^2}$$

🐆 The masses squared for the **mass eigenstates** (A^1_μ, A^2_μ, A^3_μ) are

$$m_{A^1}^2 = g_{\rm D}^2 v_{123}^2, \quad m_{A^2}^2 = \frac{g_{\rm D}^2}{2}\left(v_{123}^2 - \sqrt{v_{123}^4 - 4v_1^2 v_2^2}\right), \quad m_{A^3}^2 = \frac{g_{\rm D}^2}{2}\left(v_{123}^2 + \sqrt{v_{123}^4 - 4v_1^2 v_2^2}\right)$$

🍌 The relation between the two bases is $\tilde{A}^a_\mu = \mathcal{O}_{A,ab} A^b_\mu$

$$\mathcal{O}_A = \begin{pmatrix} 1 & & \\ & c_\theta & -s_\theta \\ & s_\theta & c_\theta \end{pmatrix}, \quad s_\theta \equiv \sin\theta = \frac{\sqrt{2} v_2 v_3}{\sqrt{v_{123}^4 - 4v_1^2 v_2^2 + (v_2^2 - v_1^2 - v_3^2)\sqrt{v_{123}^4 - 4v_1^2 v_2^2}}}, \quad c_\theta \equiv \cos\theta$$

🍊 Nonzero v_1, v_2, and v_3 👉 **No degeneracy** in the mass eigenstates

🍓 Mass hierarchy $m_{A^2} \leq m_{A^3} \leq m_{A^1}$ 🍏 **$P'_{\rm D}$-even** A^1_μ 🍎 **$P'_{\rm D}$-odd** A^2_μ, A^3_μ

🎊 The **lightest gauge boson** A^2 is a **stable vector DM candidate**

Dark Goldstone and Higgs Bosons

- The corresponding **dark Goldstone bosons** eaten by A^1, A^2, and A^3 are
  - P'_D**-even** $G_1 = v_{123}^{-1}(v_1\phi_2 + v_3\chi_2 - v_2\chi_3)$
  - P'_D**-odd** $G_2 = (c_\theta^2 v_1^2 + s_\theta^2 v_2^2 + c_\theta^2 v_3^2)^{-1/2}[-c_\theta v_1\phi_1 + (s_\theta v_2 - c_\theta v_3)\chi_1]$
  - P'_D**-odd** $G_3 = (s_\theta^2 v_1^2 + c_\theta^2 v_2^2 + s_\theta^2 v_3^2)^{-1/2}[s_\theta v_1\phi_1 + (c_\theta v_2 + s_\theta v_3)\chi_1]$
- **Dark Higgs bosons** orthogonal to these Goldstone bosons can be chosen as

$$\tilde{h}_1 = v_{23}^{-1}(v_2\chi_2 + v_3\chi_3), \quad \tilde{h}_2 = (v_{23}v_{123})^{-1}(v_{23}^2\phi_2 - v_1v_3\chi_2 + v_1v_2\chi_3), \quad \tilde{h}_3 = \phi_3$$

- These Higgs bosons mix with the SM one $\tilde{h}_0$, and the mass-squared matrix $\mathcal{M}_h^2$ for $(\tilde{h}_0,\ \tilde{h}_1,\ \tilde{h}_2,\ \tilde{h}_3)$ can be diagonalized by an orthogonal matrix $\mathcal{O}_h$:

$$\mathcal{O}_h^\mathrm{T}\mathcal{M}_h^2\mathcal{O}_h = \mathrm{diag}(m_{h_0}^2,\ m_{h_1}^2,\ m_{h_2}^2,\ m_{h_3}^2)$$

- The **Higgs mass eigenstates** $(h_0,\ h_1,\ h_2,\ h_3)$ are defined by $\tilde{h}_i = \mathcal{O}_{h,ij}h_j$
- We require h_0 to be the **SM-like Higgs boson** which receives the most contribution from $\tilde{h}_0$, and adopt a mass hierarchy convention $m_{h_1} \leq m_{h_2} \leq m_{h_3}$

DM-nucleon Scattering

🌸 **Higgs-portal interactions** $\mathcal{L}_{\text{portal}} = \sum_{i=0}^{3} \left(\frac{\kappa_i v_0}{2} h_i A_\mu^2 A^{2,\mu} - \sum_f \frac{\mathcal{O}_{h,0i} m_f}{v_0} h_i \bar{f} f \right)$

$$\kappa_i = \frac{2g_D^2}{v_0} \left\{ \frac{\mathcal{O}_{h,1i}}{v_{23}} (s_\theta v_2 - c_\theta v_3)^2 - \frac{\mathcal{O}_{h,2i} v_1 v_2}{v_{23} v_{123}} (s_{2\theta} v_2 - c_{2\theta} v_3) + \mathcal{O}_{h,3i} c_\theta^2 v_1 \right\}$$

🌲 Spin-independent (SI) A^2-**nucleon scattering** cross section

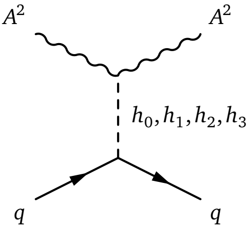
$$\sigma_N^{\text{SI}} = \frac{G_{A^2N}^2 \mu_{A^2,N}^2}{4\pi m_{A^2}^2}, \quad \mu_{A^2,N} = \frac{m_{A^2} m_N}{m_{A^2} + m_N}, \quad G_{A^2N} = -m_N \sum_q f_q^N \sum_{i=0}^{3} \frac{\kappa_i \mathcal{O}_{h,0i}}{m_{h_i}^2}$$

🍁 f_q^N are the nucleon form factors for quarks

🍉 The scattering cross section σ_N^{SI} is constrained by the **XENON1T direct detection** experiment

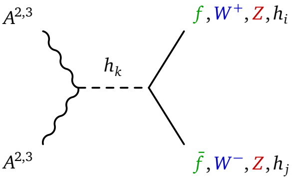
[XENON Coll., 1805.12562, PRL]

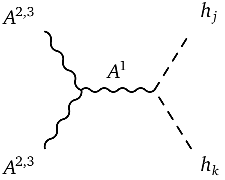
🍈 The future **LZ direct detection** experiment could improve the sensitivity [Mount *et al.*, 1703.09144]



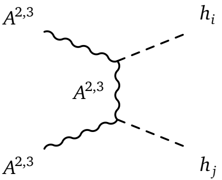
DM Annihilation

- The DM relic density is determined by **DM annihilation** in the early Universe

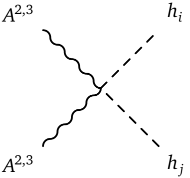




- The **coannihilation effect** between the P'_{D}**-odd** A^2_μ and A^3_μ would be significant if their masses are close



- We utilize `micrOMEGAs` to evaluate the freeze-out effective annihilation cross section $\langle \sigma_{\mathrm{ann}} v \rangle_{\mathrm{FO}}$ and the relic density $\Omega_{\mathrm{DM}} h^2$ including the coannihilation effect



- In the present Universe, $A^2 A^2$ **annihilation** basically occurs in the low-velocity limit, and the corresponding cross section $\langle \sigma_{\mathrm{ann}} v \rangle_0$ is constrained by the **Fermi-LAT γ-ray observations** of 27 dwarf galaxies for 11 years

[Hoof, Geringer-Sameth & Trotta, 1812.06986, JCAP]

Random Parameter Scan

🌱 Free parameters: $g_{\mathrm{D}},\ \lambda_0,\ \lambda_1,\ \lambda_2,\ \lambda_3,\ \lambda_4,\ \lambda_5,\ \lambda_6,\ \lambda_{10},\ \lambda_{20},\ \lambda_{30},\ v_1,\ v_2,\ v_3$

🌿 **Random scan** in logarithmic scales within the following ranges

$$10^{-3} < g_{\mathrm{D}},\ \lambda_0,\ \lambda_1,\ \lambda_2,\ |\lambda_3|,\ |\lambda_4|,\ |\lambda_5|,\ |\lambda_6|,\ |\lambda_{10}|,\ |\lambda_{20}|,\ |\lambda_{30}| < 1$$

$$10\ \mathrm{GeV} < v_1,\ v_2,\ v_3 < 10^3\ \mathrm{GeV}$$

🐼 Require the **SM-like Higgs mass** m_{h_0} lying within the 3σ range of the measured value 125.10 ± 0.14 GeV [PDG 2020]

🐮 The SM-like Higgs boson h_0 is further tested at 95% C.L. by `Lilith` based on **current LHC Higgs measurements** [Kraml *et al.*, 1908.03952, SciPost Phys.]

🦁 The **exotic Higgs bosons** h_1, h_2, and h_3 should pass the constraints from **direct searches** at the LEP and the LHC [Falkowski *et al.*, 1502.01361, JHEP]

🐯 The deviations of the **electroweak precision observables** Γ_Z, R_ℓ, R_b, m_W, and $\sin\theta^\ell_{\mathrm{eff}}$ due to **one-loop corrections** of the exotic Higgs bosons should be within the 2σ ranges of the experimental values [PDG 2020]

🐲 Require the predicted **DM relic density** $\Omega_{\mathrm{DM}}h^2$ lying within the 3σ range of the Planck measured value 0.1200 ± 0.0012 [Planck coll., 1807.06209, A&A]

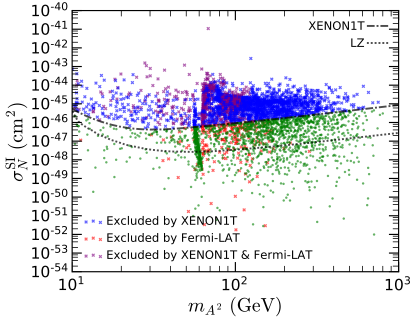
Blue points: excluded by XENON1T

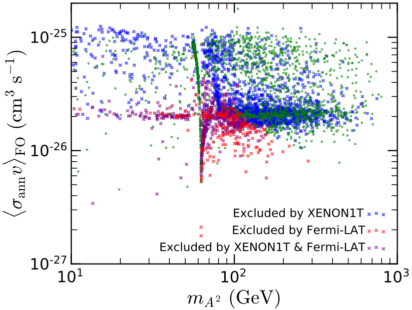
Red points: excluded by Fermi-LAT

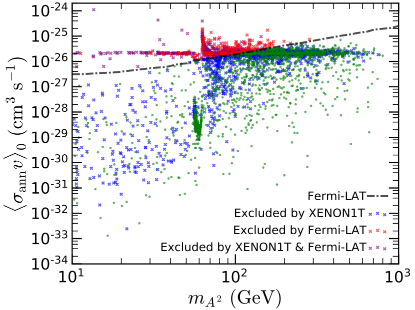
Purple points: excluded by both XENON1T & Fermi-LAT

Green points survive from all bounds

The $h_0/h_1/h_2$ **resonance effects** and h_1h_1 **threshold effects** could result in **nonstandard values** of $\langle\sigma_{\rm ann}v\rangle_{\rm FO}$

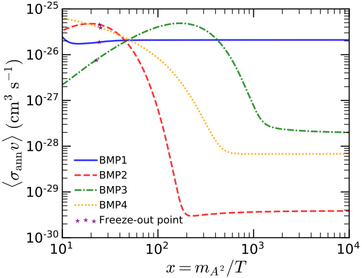






Benchmark Points (BMPs)

- **BMP1**: standard (sd)
  $\langle\sigma_{\text{ann}}v\rangle_{\text{FO}} \simeq \langle\sigma_{\text{ann}}v\rangle_{\text{sd}}$
- **BMP2**: h_1 resonance
  $\langle\sigma_{\text{ann}}v\rangle_{\text{FO}} > \langle\sigma_{\text{ann}}v\rangle_{\text{sd}}$
- **BMP3**: h_0 resonance
  $\langle\sigma_{\text{ann}}v\rangle_{\text{FO}} < \langle\sigma_{\text{ann}}v\rangle_{\text{sd}}$
- **BMP4**: $h_1 h_1$ threshold
  $\langle\sigma_{\text{ann}}v\rangle_{\text{FO}} > \langle\sigma_{\text{ann}}v\rangle_{\text{sd}}$



	BMP1	BMP2	BMP3	BMP4
g_D	0.232	0.392	0.190	0.293
λ_0	0.130	0.171	0.129	0.128
λ_1	0.112	0.757	0.0134	0.431
λ_2	0.0631	0.0830	0.0312	0.0103
λ_3	0.00144	−0.00810	0.00362	0.00877
λ_4	0.00654	−0.0367	−0.0228	−0.0616
λ_5	0.00795	−0.0207	−0.0200	0.00587
λ_6	0.00177	0.0414	0.136	0.578
λ_{10}	0.0124	0.0353	−0.00189	−0.0574
λ_{20}	0.00105	−0.108	−0.00107	0.0024
λ_{30}	0.00117	0.00371	−0.0115	0.00621
v_1 (GeV)	714	179	692	973
v_2 (GeV)	647	485	353	410
v_3 (GeV)	35.3	12.0	247	204
m_{A^1} (GeV)	224	203	155	315
m_{A^2} (GeV)	149	**70.2**	**62.2**	**117**
m_{A^3} (GeV)	167	190	142	293
m_{h_1} (GeV)	51.3	**147**	182	**118**
m_{h_2} (GeV)	462	402	215	1140
m_{h_3} (GeV)	676	441	412	1810
$\frac{\langle\sigma_{\text{ann}}v\rangle_{\text{FO}}}{\text{cm}^3/\text{s}}$	1.88×10^{-26}	4.52×10^{-26}	7.55×10^{-27}	3.87×10^{-26}
$\frac{\langle\sigma_{\text{ann}}v\rangle_0}{\text{cm}^3/\text{s}}$	2.10×10^{-26}	3.89×10^{-30}	1.93×10^{-28}	6.96×10^{-29}
σ_N^{SI} (cm^2)	2.02×10^{-47}	1.41×10^{-47}	1.04×10^{-50}	8.58×10^{-47}
$\Omega_{\text{DM}}h^2$	0.122	0.118	0.117	0.117

Generalization to Arbitrary $\mathrm{SO}(N)_\mathrm{D}$ Cases

- Our vector DM setup for a dark $SU(2)_\mathrm{D} \simeq SO(3)_\mathrm{D}$ gauge theory can be generalized to a dark $\mathrm{SO}(N)_\mathrm{D}$ $(N > 3)$ **gauge theory**
- Introduce $N-1$ **real Higgs multiplets** in the N-dimensional **fundamental representation** to completely break the $\mathrm{SO}(N)_\mathrm{D}$ gauge symmetry
- We can prove that all renormalizable terms in the Lagrangian are invariant under a **dark parity** $P_\mathrm{D} \in \mathrm{O}(N)_\mathrm{D}$ with $\det(P_\mathrm{D}) = -1$
- The Lagrangian accidentally respects a **global** $\mathrm{O}(N)_\mathrm{D}$ **symmetry**
- All the $N-1$ linearly independent VEVs of the Higgs multiplets determine a $(N-1)$**-dimensional hypersurface** in the representation space
- The **reflection** $P'_\mathrm{D} \in \mathrm{O}(N)_\mathrm{D}$ with respect to **this hypersurface** indicates a **remaining** Z'_2 **symmetry** that ensures the stability of the **lightest** P'_D**-odd** dark gauge boson, which serves as a vector DM candidate
- $N(N-1)/2$ gauge fields A^μ_{ab} $(a, b = 1, 2, \cdots N)$ satisfying $A^\mu_{ab} = -A^\mu_{ba}$
- $N-1$ fields A^μ_{a1} $(a > 1)$ are P'_D**-odd**
- The other A^μ_{ab} $(a \neq b)$ are P'_D**-even**

Summary

- We propose a **vector DM** model with a **dark** $\mathrm{SU}(2)_{\mathrm{D}}$ **gauge symmetry** that is spontaneously broken by **two real** $\mathrm{SU}(2)_{\mathrm{D}}$ **Higgs triplets**
- All dark gauge bosons become massive, and the lightest one is a vector DM candidate whose stability is guaranteed by a **remaining** Z_2' **symmetry**
- We study the parameter space constrained by the **Higgs** and **electroweak measurements**, **exotic Higgs searches**, the **DM relic density**, and **direct** and **indirect detection** experiments
- We prove that the similar methodology can be used to construct vector DM models from an **arbitrary** $\mathrm{SO}(N)$ **gauge group**

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Thanks for your attention!

Mass-squared Matrix for the Neutral Higgs Bosons

Minimization conditions for the potential give

$$\mu_0^2 = \lambda_0 v_0^2 + \lambda_{10} v_1^2 + \lambda_{20} v_{23}^2 + \lambda_{30} v_1 v_3$$

$$\mu_1^2 = 2\lambda_1 v_1^2 + \lambda_3 v_{23}^2 + \lambda_4 v_1 v_3 + \frac{1}{2}\lambda_{10} v_0^2$$

$$\mu_2^2 = 2\lambda_2 v_{23}^2 + \lambda_3 v_1^2 + \lambda_5 v_1 v_3 + \frac{1}{2}\lambda_{20} v_0^2$$

$$\mu_3^2 = \lambda_4 v_1^2 + \lambda_5 v_{23}^2 + 2\lambda_6 v_1 v_3 + \frac{1}{2}\lambda_{30} v_0^2$$

Mass-squared matrix for the **neutral Higgs bosons** $(\tilde{h}_0,\ \tilde{h}_1,\ \tilde{h}_2,\ \tilde{h}_3)$

$$\mathcal{M}_h^2 = \begin{pmatrix} 2\lambda_0 v_0^2 & 2\lambda_{20} v_0 v_{23} + \frac{\lambda_{30} v_0 v_1 v_3}{v_{23}} & \frac{\lambda_{30} v_0 v_2 v_{123}}{v_{23}} & 2\lambda_{10} v_0 v_1 + \lambda_{30} v_0 v_3 \\ * & 8\lambda_2 v_{23}^2 + 4\lambda_5 v_1 v_3 + \frac{2\lambda_6 v_1^2 v_3^2}{v_{23}^2} & 2\lambda_5 v_2 v_{123} + \frac{2\lambda_6 v_1 v_2 v_3 v_{123}}{v_{23}^2} & 4\lambda_3 v_1 v_{23} + 2\lambda_5 v_3 v_{23} + \frac{2 v_1 v_3 (\lambda_4 v_1 + \lambda_6 v_3)}{v_{23}} \\ * & * & \frac{2\lambda_6 v_2^2 v_{123}^2}{v_{23}^2} & \frac{2\lambda_4 v_1 v_2 v_{123}}{v_{23}} + \frac{2\lambda_6 v_2 v_3 v_{123}}{v_{23}} \\ * & * & * & 8\lambda_1 v_1^2 + 4\lambda_4 v_1 v_3 + 2\lambda_6 v_3^2 \end{pmatrix}$$

Corrections to Electroweak Gauge Boson Self-energies

☘ The shifts to the $g^{\mu\nu}$ coefficients of the **electroweak gauge boson** vacuum polarization amplitudes contributed by **one-loop corrections** from the **exotic Higgs bosons** are given by

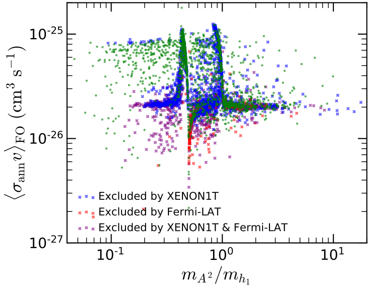
$$\delta\Pi_{\gamma\gamma}(p^2) = \delta\Pi_{Z\gamma}(p^2) = 0$$

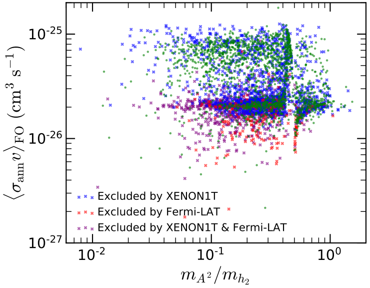
$$\delta\Pi_{WW}(p^2) = \frac{m_W^2}{4\pi^2 v_0^2}\left\{\sum_{i=0}^{3}\mathcal{O}_{h,i0}^2\left[\frac{m_{h_i}^2}{4}\ln m_{h_i}^2 + F(p^2, m_W^2, m_{h_i}^2)\right] - \frac{m_{h_{\rm SM}}^2}{4}\ln m_{h_{\rm SM}}^2 - F(p^2, m_W^2, m_{h_{\rm SM}}^2)\right\}$$

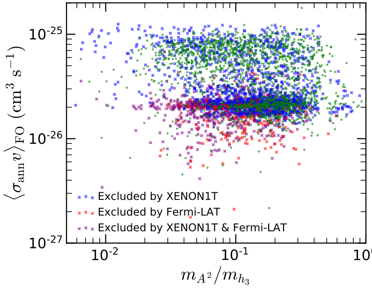
$$\delta\Pi_{ZZ}(p^2) = \frac{m_Z^2}{4\pi^2 v_0^2}\left\{\sum_{i=0}^{3}\mathcal{O}_{h,i0}^2\left[\frac{m_{h_i}^2}{4}\ln m_{h_i}^2 + F(p^2, m_Z^2, m_{h_i}^2)\right] - \frac{m_{h_{\rm SM}}^2}{4}\ln m_{h_{\rm SM}}^2 - F(p^2, m_Z^2, m_{h_{\rm SM}}^2)\right\}$$

☘ $F(p^2, m_1^2, m_2^2) = \int_0^1 dx \left(m_1^2 - \frac{\Delta}{2}\right)\ln\Delta, \quad \Delta = x m_2^2 + (1-x)m_1^2 - p^2 x(1-x)$

$\langle\sigma_{\rm ann}v\rangle_{\rm FO}$ versus m_{A^2}/m_{h_i}







Comparison between $\langle \sigma_{\text{ann}} v \rangle_{\text{FO}}$ and $\langle \sigma_{\text{ann}} v \rangle_0$