Pseudo-Nambu-Goldstone Dark Matter, Two Higgs doublets, and Gravitational Waves

Zhao-Huan Yu（余钊焕）

School of Physics, Sun Yat-Sen University

https://yuzhaohuan.gitee.io

Based on Xue-Min Jiang, Chengfeng Cai, Zhao-Huan Yu, Yu-Pan Zeng, Hong-Hao Zhang, 1907.09684, PRD

Zhao Zhang, Chengfeng Cai, Xue-Min Jiang, Yi-Lei Tang, Zhao-Huan Yu, Hong-Hao Zhang, 2102.01588, accepted by JHEP

Jilin University, Changchun

May 10, 2021

Thermal Dark Matter

Conventionally, **dark matter (DM)** is assumed to be a **thermal relic** remaining from the early Universe

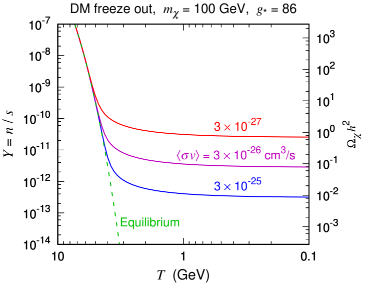
DM relic abundance observation

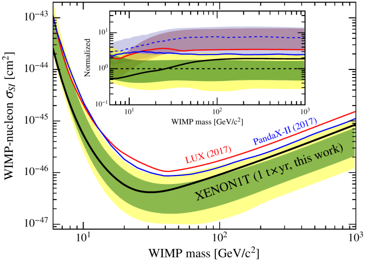
Particle mass $m_\chi \sim \mathcal{O}(\text{GeV}) - \mathcal{O}(\text{TeV})$

Interaction strength $\sim$ **weak strength**

"Weakly interacting massive particles"

"WIMPs"





[XENON Coll., 1805.12562, PRL]

Direct detection for WIMPs

No robust signal found so far

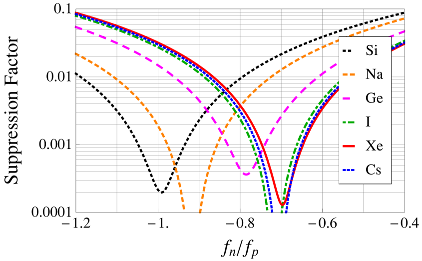
Great challenge to the thermal dark matter paradigm

Save the Thermal DM Paradigm

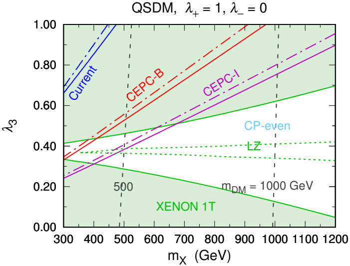
- **Enhance DM annihilation** at the **freeze-out epoch**
- **Coannihilation, resonance effect, Sommerfeld enhancement, *etc.***
- **Suppress DM-nucleon scattering** at **zero momentum transfer**
- **Isospin-violating interactions with protons and neutrons**

  Feng *et al.*, 1102.4331 PLB; Frandsen *et al.*, 1107.2118, JHEP; ···
- **"Blind spots": particular parameter values lead to suppression**

  Cheung *et al.*, 1211.4873, JHEP; Cai, **ZHY**, Zhang, 1705.07921, NPB; Han *et al.*, 1810.04679, JHEP; Altmannshofer, *et al.*, 1907.01726, PRD; ···



[Frandsen *et al.*, 1107.2118, JHEP]



[Cai, **ZHY**, Zhang, 1705.07921, NPB]

Save the Thermal DM Paradigm

Suppress DM-nucleon scattering at zero momentum transfer

- **Mediated by pseudoscalars: velocity-dependent SD scattering**
  Ipek *et al.*, 1404.3716, PRD; Berlin *et al.*, 1502.06000, PRD;
  Goncalves, *et al.*, 1611.04593, PRD; Bauer, *et al.*, 1701.07427, JHEP; ⋯

- **Relevant DM couplings vanish due to special symmetries**
  Dedes & Karamitros, 1403.7744, PRD; Tait & **ZHY**, 1601.01354, JHEP;
  Cai, **ZHY**, Zhang, 1611.02186, NPB; ⋯

Triplet-quadruplet fermionic DM model

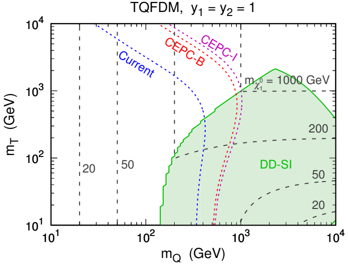
Custodial symmetry limit $y_1 = y_2$

👇

DM couplings to h and Z vanish for $m_Q < m_T$

👇

DM-nucleon scattering vanishes at tree level



[Cai, **ZHY**, Zhang, 1611.02186, NPB]

- **DM particle is a pseudo-Nambu-Goldstone boson (pNGB) protected by an approximate global symmetry** [Gross, Lebedev, Toma, 1708.02253, PRL]

pNGB Dark Matter [Gross, Lebedev, Toma, 1708.02253, PRL]

- Standard model (SM) Higgs doublet H, **complex scalar S** (SM singlet)
- Scalar potential respects a **softly broken global U(1) symmetry** $S \to e^{i\alpha} S$
- U(1) **symmetric** $V_0 = -\frac{\mu_H^2}{2}|H|^2 - \frac{\mu_S^2}{2}|S|^2 + \frac{\lambda_H}{2}|H|^4 + \frac{\lambda_S}{2}|S|^4 + \lambda_{HS}|H|^2|S|^2$
- **Soft breaking** $V_{\text{soft}} = -\frac{\mu_S'^2}{4} S^2 + \text{H.c.}$
- Soft breaking parameter $\mu_S'^2$ can be made **real** and **positive** by redefining S
- V_{soft} can be justified by treating $\mu_S'^2$ as a **spurion** from an underlying theory
- H and S develop vacuum expectation values (VEVs) v and v_s

$$H \to \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h \end{pmatrix}, \quad S = \frac{1}{\sqrt{2}}(v_s + s + i\chi)$$

- The soft breaking term V_{soft} give a mass to χ: $m_\chi = \mu_S'$
- χ is a **stable pNGB**, acting as a **DM candidate**

Scalar Mixing and Interactions [Gross, Lebedev, Toma, 1708.02253, PRL]

Mixing of the **CP-even Higgs bosons** h and s

$$\mathcal{M}^2_{h,s} = \begin{pmatrix} \lambda_H v^2 & \lambda_{HS} v v_s \\ \lambda_{HS} v v_s & \lambda_S v_s^2 \end{pmatrix}, \quad O^{\mathrm{T}} \mathcal{M}^2 O = \begin{pmatrix} m^2_{h_1} & \\ & m^2_{h_2} \end{pmatrix}$$

$$O = \begin{pmatrix} c_\theta & s_\theta \\ -s_\theta & c_\theta \end{pmatrix}, \quad c_\theta \equiv \cos\theta, \quad s_\theta \equiv \sin\theta, \quad \tan 2\theta = \frac{2\lambda_{HS} v v_s}{\lambda_S v_s^2 - \lambda_H v^2}$$

$$\begin{pmatrix} h \\ s \end{pmatrix} = O \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}, \quad m^2_{h_1,h_2} = \frac{1}{2}\left(\lambda_H v^2 + \lambda_S v_s^2 \mp \frac{\lambda_S v_s^2 - \lambda_H v^2}{\cos 2\theta} \right)$$

Higgs portal interactions

$$\mathcal{L} \supset -\frac{\lambda_{HS} v}{2} h \chi^2 - \frac{\lambda_S v_s}{2} s \chi^2 - \sum_f \frac{m_f}{v} h \bar{f} f$$

$$= \frac{m^2_{h_1} s_\theta}{2 v_s} h_1 \chi^2 - \frac{m^2_{h_2} c_\theta}{2 v_s} h_2 \chi^2 - \sum_f \frac{m_f}{v} (h_1 c_\theta + h_2 s_\theta) \bar{f} f$$

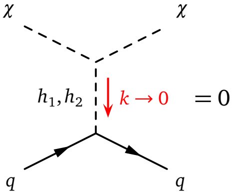
DM-nucleon Scattering [Gross, Lebedev, Toma, 1708.02253, PRL]

✨ **DM-quark** interactions induce **DM-nucleon** scattering in direct detection

✏️ **DM-quark scattering amplitude** from Higgs portal interactions

$$\mathcal{M}(\chi q \to \chi q) \propto \frac{m_q s_\theta c_\theta}{v v_s} \left(\frac{m_{h_1}^2}{t - m_{h_1}^2} - \frac{m_{h_2}^2}{t - m_{h_2}^2} \right)$$

$$= \frac{m_q s_\theta c_\theta}{v v_s} \frac{t(m_{h_1}^2 - m_{h_2}^2)}{(t - m_{h_1}^2)(t - m_{h_2}^2)}$$



🔥 **Zero momentum transfer limit** $t = k^2 \to 0, \quad \mathcal{M}(\chi q \to \chi q) \to 0$

👉 DM-nucleon scattering cross section **vanishes** at tree level

💡 Tree-level interactions of a **pNGB** are generally **momentum suppressed**

⛅ **One-loop corrections** typically lead to $\sigma_{\chi N}^{\mathrm{SI}} \lesssim \mathcal{O}(10^{-50})\ \mathrm{cm}^2$

[Azevedo *et al.*, 1810.06105, JHEP; Ishiwata & Toma, 1810.08139, JHEP]

👉 **Beyond capability** of current and near future direct detection experiments

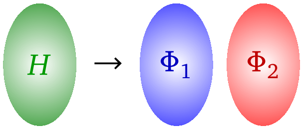
Generalizations

- Generalize the softly broken global U(1) to $O(N)$, $SU(N)$ or $U(1) \times S_N$

  [Alanne *et al.*, 1812.05996, PRD; Karamitros, 1901.09751, PRD]

- Multiple pNGBs constituting **multi-component** dark matter

- We extend the study to **two-Higgs-doublet models (2HDMs)**



- ❓ Does **DM-nucleon scattering** still **vanish** at zero momentum transfer?

- ❓ How do current **Higgs measurements** in the LHC experiments constrain such a model?

- ❓ Can the observed **relic abundance** be obtained via the thermal mechanism?

- ❓ How are the constraints from **indirect detection**?

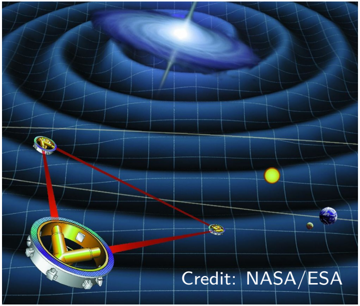
Gravitational Waves from a First-order Phase Transition

🔭 The discovery of **gravitational waves (GWs)** by LIGO in 2015 opens a new window to new physics models

💡 Introducing new scalar fields may change the **electroweak phase transition** to be a **first-order phase transition (FOPT)**

💥 A cosmological FOPT could induce a **stochastic GW background** with $f \sim \mathrm{mHz}$

👉 Potential signals in **future space-based GW interferometers** like **LISA**, **TianQin**, **Taiji**, **DECIGO**, and **BBO**



Credit: NASA/ESA

😟 However, the **original pNGB model** can only results in **second-order** phase transitions

[Kannike & Raidal, 1901.03333, PRD]

😎 We will try to explore FOPTs and the resulting GW signals in the **2HDM extension**

pNBG DM and Two Higgs Doublets

- **Two Higgs doublets** Φ_1 and Φ_2 with $Y = 1/2$, **complex scalar singlet** S
- Scalar potential respects a softly broken global U(1) symmetry $S \to e^{i\alpha} S$
- Two **common assumptions** for 2HDMs
  - CP is conserved in the scalar sector
  - There is a Z_2 symmetry $\Phi_1 \to -\Phi_1$ or $\Phi_2 \to -\Phi_2$ forbidding quartic terms that are odd in Φ_1 or Φ_2, but it can be softly broken by quadratic terms
- Scalar potential constructed with Φ_1 and Φ_2

$$V_1 = m_{11}^2|\Phi_1|^2 + m_{22}^2|\Phi_2|^2 - m_{12}^2(\Phi_1^\dagger\Phi_2 + \Phi_2^\dagger\Phi_1) + \frac{\lambda_1}{2}|\Phi_1|^4 + \frac{\lambda_2}{2}|\Phi_2|^4$$
$$+ \lambda_3|\Phi_1|^2|\Phi_2|^2 + \lambda_4|\Phi_1^\dagger\Phi_2|^2 + \frac{\lambda_5}{2}[(\Phi_1^\dagger\Phi_2)^2 + (\Phi_2^\dagger\Phi_1)^2]$$

- U(1) **symmetric** potential terms involving S

$$V_2 = -m_S^2|S|^2 + \frac{\lambda_S}{2}|S|^4 + \kappa_1|\Phi_1|^2|S|^2 + \kappa_2|\Phi_2|^2|S|^2$$

- Quadratic term **softly breaking** the global U(1): $V_{\text{soft}} = -\dfrac{m_S'^2}{4}S^2 + \text{H.c.}$

Scalars

- Φ_1, Φ_2, and S develop VEVs v_1, v_2 and v_s

$$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ (v_1 + \rho_1 + i\eta_1)/\sqrt{2} \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_2^+ \\ (v_2 + \rho_2 + i\eta_2)/\sqrt{2} \end{pmatrix}, \quad S = \frac{v_s + s + i\chi}{\sqrt{2}}$$

- χ is a **stable pNGB** with $m_\chi = m_S'$, acting as a **DM candidate**
- Mass terms for **charged scalars** and CP**-odd scalars**

$$-\mathcal{L}_{\text{mass}} \supset \left[m_{12}^2 - \frac{1}{2}(\lambda_4 + \lambda_5)v_1 v_2\right](\phi_1^-, \quad \phi_2^-)\begin{pmatrix} v_2/v_1 & -1 \\ -1 & v_1/v_2 \end{pmatrix}\begin{pmatrix} \phi_1^+ \\ \phi_2^+ \end{pmatrix}$$
$$+ \frac{1}{2}(m_{12}^2 - \lambda_5 v_1 v_2)(\eta_1, \quad \eta_2)\begin{pmatrix} v_2/v_1 & -1 \\ -1 & v_1/v_2 \end{pmatrix}\begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix}$$

$$\begin{pmatrix} \phi_1^+ \\ \phi_2^+ \end{pmatrix} = R(\beta)\begin{pmatrix} G^+ \\ H^+ \end{pmatrix}, \quad \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} = R(\beta)\begin{pmatrix} G^0 \\ a \end{pmatrix}, \quad R(\beta) = \begin{pmatrix} c_\beta & -s_\beta \\ s_\beta & c_\beta \end{pmatrix}, \quad \tan\beta = \frac{v_2}{v_1}$$

- $G^\pm$ and G^0 are **massless Nambu-Goldstone bosons** eaten by $W^\pm$ and Z
- $H^\pm$ and a are **physical states**

$$m_{H^+}^2 = \frac{v_1^2 + v_2^2}{v_1 v_2}\left[m_{12}^2 - \frac{1}{2}(\lambda_4 + \lambda_5)v_1 v_2\right], \quad m_a^2 = \frac{v_1^2 + v_2^2}{v_1 v_2}(m_{12}^2 - \lambda_5 v_1 v_2)$$

CP-even Scalars and Weak Gauge Bosons

Mass terms for **CP-even scalars** $-\mathcal{L}_{\text{mass}} \supset \frac{1}{2}(\rho_1, \quad \rho_2, \quad s)\mathcal{M}^2_{\rho s}\begin{pmatrix}\rho_1\\ \rho_2\\ s\end{pmatrix}$

$$\mathcal{M}^2_{\rho s} = \begin{pmatrix} \lambda_1 v_1^2 + m_{12}^2 \tan\beta & \lambda_{345} v_1 v_2 - m_{12}^2 & \kappa_1 v_1 v_s \\ \lambda_{345} v_1 v_2 - m_{12}^2 & \lambda_2 v_2^2 + m_{12}^2 \cot\beta & \kappa_2 v_2 v_s \\ \kappa_1 v_1 v_s & \kappa_2 v_2 v_s & \lambda_S v_s^2 \end{pmatrix}, \quad \lambda_{345} \equiv \lambda_3 + \lambda_4 + \lambda_5$$

$$\begin{pmatrix}\rho_1\\ \rho_2\\ s\end{pmatrix} = O \begin{pmatrix}h_1\\ h_2\\ h_3\end{pmatrix}, \quad O^{\mathrm{T}}\mathcal{M}^2_{\rho s}O = \mathrm{diag}(m^2_{h_1}, m^2_{h_2}, m^2_{h_3}), \quad m_{h_1} \le m_{h_2} \le m_{h_3}$$

One of h_i should behave like the **125 GeV SM Higgs boson**

Mass terms for **weak gauge bosons**

$$-\mathcal{L}_{\text{mass}} \supset \frac{g^2}{4}(v_1^2 + v_2^2) W^{-,\mu} W^+_\mu + \frac{1}{2}\frac{g^2}{4c_{\mathrm{W}}^2}(v_1^2 + v_2^2) Z^\mu Z_\mu, \quad c_{\mathrm{W}} \equiv \cos\theta_{\mathrm{W}}$$

$$m_W = \frac{gv}{2}, \quad m_Z = \frac{gv}{2c_{\mathrm{W}}}, \quad v \equiv \sqrt{v_1^2 + v_2^2} = (\sqrt{2}G_{\mathrm{F}})^{-1/2} = 246.22\ \text{GeV}$$

Yukawa Couplings

😨 In 2HDMs, diagonalizing the **fermion mass matrix** **cannot** make sure that the **Yukawa interactions** are simultaneously diagonalized

👉 Tree-level **flavor-changing neutral currents (FCNCs)** 👉 flavor problems

😊 If all fermions with the same quantum numbers just couple to the one **same** Higgs doublet, the FCNCs will be **absent** at tree level

[Glashow & Weinberg, PRD 15, 1958 (1977); Paschos, PRD 15, 1966 (1977)]

💡 This can be achieved by assuming particular Z_2 **symmetries** for the Higgs doublets and fermions

✏️ **Four independent types** of Yukawa couplings without tree-level FCNCs

Type I: $\mathcal{L}_{\mathrm{Y,I}} = -y_{\ell_i}\bar{L}_{i\mathrm{L}}\ell_{i\mathrm{R}}\Phi_2 - \tilde{y}_d^{ij}\bar{Q}_{i\mathrm{L}}d'_{j\mathrm{R}}\Phi_2 - \tilde{y}_u^{ij}\bar{Q}_{i\mathrm{L}}u'_{j\mathrm{R}}\tilde{\Phi}_2 + \mathrm{H.c.}$

Type II: $\mathcal{L}_{\mathrm{Y,II}} = -y_{\ell_i}\bar{L}_{i\mathrm{L}}\ell_{i\mathrm{R}}\Phi_1 - \tilde{y}_d^{ij}\bar{Q}_{i\mathrm{L}}d'_{j\mathrm{R}}\Phi_1 - \tilde{y}_u^{ij}\bar{Q}_{i\mathrm{L}}u'_{j\mathrm{R}}\tilde{\Phi}_2 + \mathrm{H.c.}$

Lepton specific: $\mathcal{L}_{\mathrm{Y,L}} = -y_{\ell_i}\bar{L}_{i\mathrm{L}}\ell_{i\mathrm{R}}\Phi_1 - \tilde{y}_d^{ij}\bar{Q}_{i\mathrm{L}}d'_{j\mathrm{R}}\Phi_2 - \tilde{y}_u^{ij}\bar{Q}_{i\mathrm{L}}u'_{j\mathrm{R}}\tilde{\Phi}_2 + \mathrm{H.c.}$

Flipped: $\mathcal{L}_{\mathrm{Y,F}} = -y_{\ell_i}\bar{L}_{i\mathrm{L}}\ell_{i\mathrm{R}}\Phi_2 - \tilde{y}_d^{ij}\bar{Q}_{i\mathrm{L}}d'_{j\mathrm{R}}\Phi_1 - \tilde{y}_u^{ij}\bar{Q}_{i\mathrm{L}}u'_{j\mathrm{R}}\tilde{\Phi}_2 + \mathrm{H.c.}$

[Branco *et al.*, 1106.0034, Phys. Rept.]

Four Types of Yukawa Couplings

✏️ Yukawa interactions for the **fermion mass eigenstates**

$$\mathcal{L}_{\mathrm{Y}} = \sum_{f=\ell_j,d_j,u_j} \left[-m_f \bar{f} f - \frac{m_f}{v} \left(\sum_{i=1}^{3} \xi_{h_i}^f h_i \bar{f} f + \xi_a^f a \bar{f} i\gamma_5 f \right) \right]$$

$$- \frac{\sqrt{2}}{v} [H^+ (\xi_a^{\ell_i} m_{\ell_i} \bar{\nu}_i P_{\mathrm{R}} \ell_i + \xi_a^{d_j} m_{d_j} V_{ij} \bar{u}_i P_{\mathrm{R}} d_j + \xi_a^{u_i} m_{u_i} V_{ij} \bar{u}_i P_{\mathrm{L}} d_j) + \mathrm{H.c.}]$$

	Type I	Type II	Lepton specific	Flipped
$\xi_{h_i}^{\ell_j}$	$O_{2i}/\sin\beta$	$O_{1i}/\cos\beta$	$O_{1i}/\cos\beta$	$O_{2i}/\sin\beta$
$\xi_{h_i}^{d_j}$	$O_{2i}/\sin\beta$	$O_{1i}/\cos\beta$	$O_{2i}/\sin\beta$	$O_{1i}/\cos\beta$
$\xi_{h_i}^{u_j}$	$O_{2i}/\sin\beta$	$O_{2i}/\sin\beta$	$O_{2i}/\sin\beta$	$O_{2i}/\sin\beta$
$\xi_a^{\ell_j}$	$\cot\beta$	$-\tan\beta$	$-\tan\beta$	$\cot\beta$
$\xi_a^{d_j}$	$\cot\beta$	$-\tan\beta$	$\cot\beta$	$-\tan\beta$
$\xi_a^{u_j}$	$-\cot\beta$	$-\cot\beta$	$-\cot\beta$	$-\cot\beta$

Vanishing of DM-nucleon Scattering

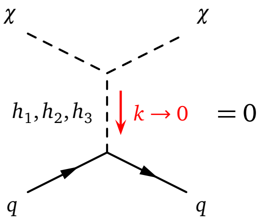
💡 Take the **type-I** Yukawa couplings as an example

⭐ **Higgs portal** interactions $\mathcal{L}_{h_i\chi^2} = \frac{1}{2}\sum_{i=1}^{3} g_{h_i\chi^2} h_i \chi^2$

$$g_{h_i\chi^2} = -\kappa_1 v_1 O_{1i} - \kappa_2 v_2 O_{2i} - \lambda_S v_s O_{3i}$$

✨ **DM-quark scattering** amplitude

$$\mathcal{M}(\chi q \to \chi q) \propto \frac{m_q}{v s_\beta}\left(\frac{g_{h_1\chi^2}O_{21}}{t-m_{h_1}^2} + \frac{g_{h_2\chi^2}O_{22}}{t-m_{h_2}^2} + \frac{g_{h_3\chi^2}O_{23}}{t-m_{h_3}^2}\right)$$



$$\xrightarrow{t\to 0} \frac{m_q}{v s_\beta}(\kappa_1 v_1,\ \kappa_2 v_2,\ \lambda_S v_s)\, O(\mathcal{M}_h^2)^{-1} O^{\mathrm{T}} \begin{pmatrix}0\\1\\0\end{pmatrix} = \frac{m_q}{v s_\beta}(\kappa_1 v_1,\ \kappa_2 v_2,\ \lambda_S v_s)\,(\mathcal{M}_{\rho s}^2)^{-1}\begin{pmatrix}0\\1\\0\end{pmatrix}$$

Interaction basis expression

$$= \frac{m_q}{v s_\beta \det(\mathcal{M}_{\rho s}^2)}(\kappa_1 v_1 \mathcal{A}_{12} + \kappa_2 v_2 \mathcal{A}_{22} + \lambda_S v_s \mathcal{A}_{32}) = 0$$

$$\mathcal{M}_h^2 \equiv \mathrm{diag}(m_{h_1}^2, m_{h_2}^2, m_{h_3}^2)$$

$$O(\mathcal{M}_h^2)^{-1}O^{\mathrm{T}} = (\mathcal{M}_{\rho s}^2)^{-1} = \frac{\mathcal{A}}{\det(\mathcal{M}_{\rho s}^2)}, \quad \mathcal{A}_{12} = -(\lambda_{345} v_1 v_2 - m_{12}^2)\lambda_S v_s^2 + \kappa_1 \kappa_2 v_1 v_2 v_s^2$$

$$\mathcal{A}_{22} = (\lambda_1 v_1^2 + m_{12}^2 \tan\beta)\lambda_S v_s^2 - \kappa_1^2 v_1^2 v_s^2, \quad \mathcal{A}_{32} = -(\lambda_1 v_1^2 + m_{12}^2 \tan\beta)\kappa_2 v_2 v_s + (\lambda_{345} v_1 v_2 - m_{12}^2)\kappa_1 v_1 v_s$$

Alignment Limit

Higgs basis → Φ_h (h) acts as the **SM Higgs doublet** (**boson**)

$$\begin{pmatrix}\Phi_h\\ \Phi_H\end{pmatrix} \equiv R^{-1}(\beta)\begin{pmatrix}\Phi_1\\ \Phi_2\end{pmatrix}, \quad \Phi_h = \begin{pmatrix}G^+\\ (v+h+iG^0)/\sqrt{2}\end{pmatrix}, \quad \Phi_H = \begin{pmatrix}H^+\\ (H+ia)/\sqrt{2}\end{pmatrix}$$

$$V_1 = m_{hh}^2|\Phi_h|^2 + m_{HH}^2|\Phi_H|^2 - m_{hH}^2(\Phi_h^\dagger\Phi_H + \Phi_H^\dagger\Phi_h) + \frac{\lambda_h}{2}|\Phi_h|^4 + \frac{\lambda_H}{2}|\Phi_H|^4 + \tilde{\lambda}_3|\Phi_h|^2|\Phi_H|^2$$
$$+ \tilde{\lambda}_4|\Phi_h^\dagger\Phi_H|^2 + \frac{1}{2}[\tilde{\lambda}_5(\Phi_h^\dagger\Phi_H)^2 + \tilde{\lambda}_6|\Phi_h|^2\Phi_H^\dagger\Phi_h + \tilde{\lambda}_7|\Phi_H|^2\Phi_h^\dagger\Phi_H + \text{H.c.}]$$

$$V_2 = -m_S^2|S|^2 + \frac{\lambda_S}{2}|S|^4 + \tilde{\kappa}_1|\Phi_h|^2|S|^2 + \tilde{\kappa}_2|\Phi_H|^2|S|^2 + \tilde{\kappa}_3(\Phi_h^\dagger\Phi_H + \Phi_H^\dagger\Phi_h)|S|^2$$

Mass-squared matrix for CP-even scalars (h, H, s)

$$\mathcal{M}_{hHs}^2 = \begin{pmatrix}\lambda_h v^2 & \tilde{\lambda}_6 v^2/2 & \tilde{\kappa}_1 v v_s\\ \tilde{\lambda}_6 v^2/2 & m_{HH}^2 + (\tilde{\lambda}_{345}v^2 + \tilde{\kappa}_2 v_s^2)/2 & \tilde{\kappa}_3 v v_s\\ \tilde{\kappa}_1 v v_s & \tilde{\kappa}_3 v v_s & \lambda_S v_s^2\end{pmatrix}$$

Alignment Limit
$$\begin{cases}\tilde{\lambda}_6 = -s_{2\beta}(c_\beta^2\lambda_1 - s_\beta^2\lambda_2) + s_{2\beta}c_{2\beta}\lambda_{345} = 0\\ \tilde{\kappa}_1 = c_\beta^2\kappa_1 + s_\beta^2\kappa_2 = 0\end{cases}$$

→ Couplings of $h_{125} = h$ to SM particles are **identical** to **SM** Higgs couplings

Parameter Scan

12 free parameters in the model

$$v_s,\ m_\chi,\ m_{12}^2,\ \tan\beta,\ \lambda_1,\ \lambda_2,\ \lambda_3,\ \lambda_4,\ \lambda_5,\ \lambda_S,\ \kappa_1,\ \kappa_2$$

Random scan within the following ranges

$$10\ \text{GeV} < v_s < 10^3\ \text{GeV},\quad 10\ \text{GeV} < m_\chi < 10^4\ \text{GeV},$$
$$(10\ \text{GeV})^2 < |m_{12}^2| < (10^3\ \text{GeV})^2,\quad 10^{-2} < \tan\beta < 10^2,$$
$$10^{-3} < \lambda_1, \lambda_2, \lambda_S < 1,\quad 10^{-3} < |\lambda_3|, |\lambda_4|, |\lambda_5|, |\kappa_1|, |\kappa_2| < 1$$

Select the parameter points satisfying **two conditions**

Positive $m_{h_{1,2,3}}^2$, $m_{H^+}^2$, and m_a^2 ☞ ensuring **physical** scalar masses

One of the CP-even Higgs bosons h_i has a mass within the 3σ range of the measured SM-like Higgs boson mass $m_h = 125.18 \pm 0.16$ GeV [PDG 2018]

☞ Recognize this scalar as the **SM-like** Higgs boson and denote it as h_{SM}

κ-framework

Couplings of the **SM-like Higgs boson** $h_{\rm SM}$ to SM particles

$$\mathcal{L}_{h_{\rm SM}} = \kappa_W g m_W h_{\rm SM} W^+_\mu W^{-,\mu} + \kappa_Z \frac{g m_Z}{2c_{\rm W}} h_{\rm SM} Z_\mu Z^\mu - \sum_f \kappa_f \frac{m_f}{v} h_{\rm SM} \bar{f} f$$
$$+ \kappa_g g^{\rm SM}_{hgg} h_{\rm SM} G^a_{\mu\nu} G^{a\mu\nu} + \kappa_\gamma g^{\rm SM}_{h\gamma\gamma} h_{\rm SM} A_{\mu\nu} A^{\mu\nu} + \kappa_{Z\gamma} g^{\rm SM}_{hZ\gamma} h_{\rm SM} A_{\mu\nu} Z^{\mu\nu}$$

$g^{\rm SM}_{hgg}$, $g^{\rm SM}_{h\gamma\gamma}$, and $g^{\rm SM}_{hZ\gamma}$ are **loop-induced** effective couplings in the SM

Modifier for the $h_{\rm SM}$ decay width $\kappa_H^2 \equiv \dfrac{\Gamma_{h_{\rm SM}} - \Gamma^{\rm BSM}_{h_{\rm SM}}}{\Gamma^{\rm SM}_h}$, $\quad \Gamma^{\rm BSM}_{h_{\rm SM}} = \Gamma^{\rm inv}_{h_{\rm SM}} + \Gamma^{\rm und}_{h_{\rm SM}}$

$\Gamma^{\rm inv}_{h_{\rm SM}}$ is the decay width into **invisible** final states, *e.g.*, $\chi\chi$

$\Gamma^{\rm und}_{h_{\rm SM}}$ is the decay width into **undetected** beyond-the-SM (BSM) final states, *e.g.*, aa, H^+H^-, $h_i h_j$, aZ, and $H^\pm W^\mp$

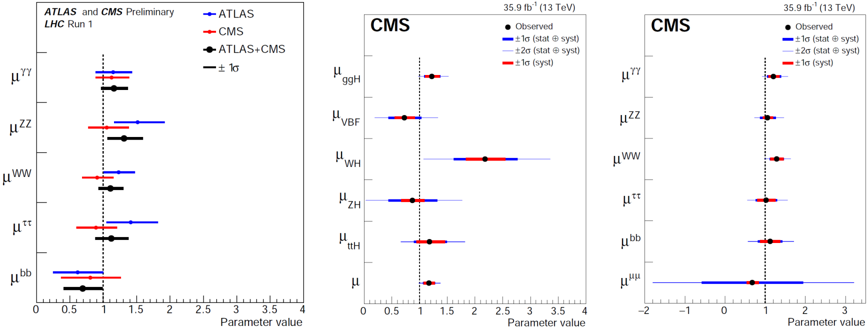
In the SM, $\kappa_W = \kappa_Z = \kappa_f = \kappa_g = \kappa_\gamma = \kappa_{Z\gamma} = \kappa_H = 1$

In our model, assuming $h_{\rm SM} = h_i$ and **type-I** Yukawa couplings,

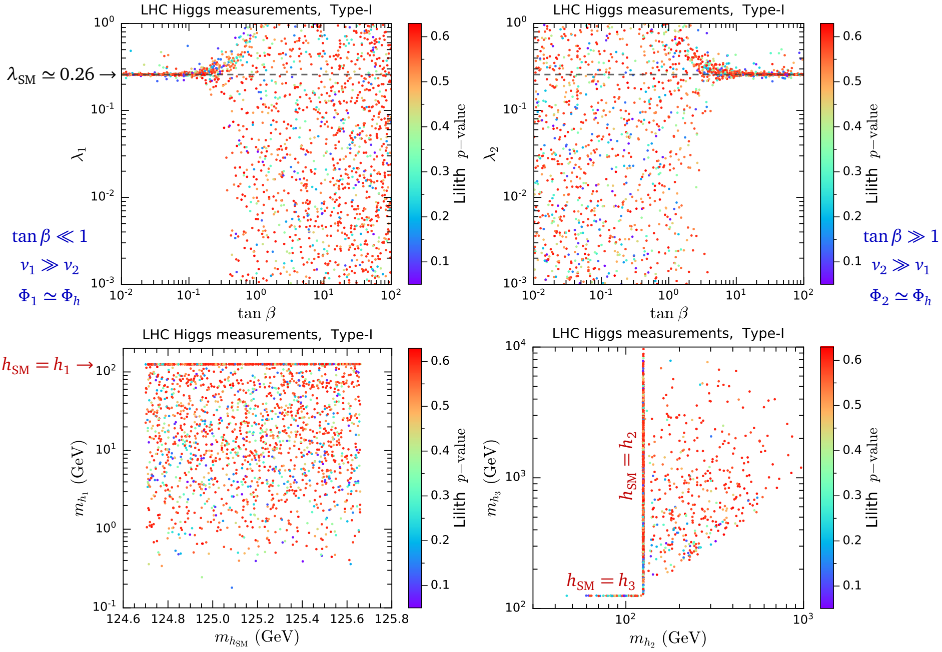
$$\kappa_Z = \kappa_W \equiv \kappa_V = c_\beta O_{1i} + s_\beta O_{2i}, \quad \kappa_f = O_{2i}/s_\beta$$

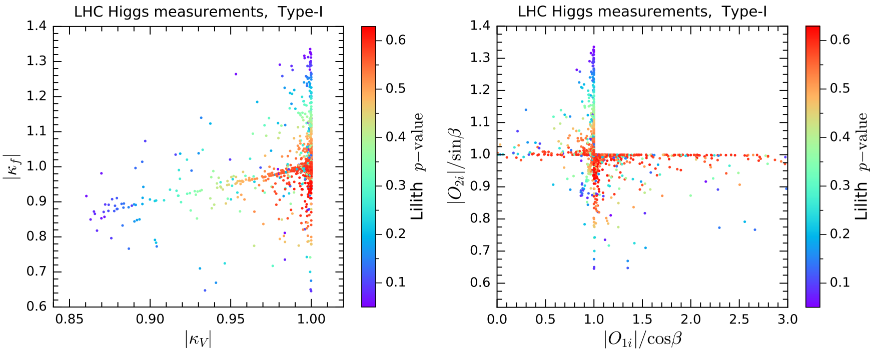
Global Fit with Higgs Measurement Data

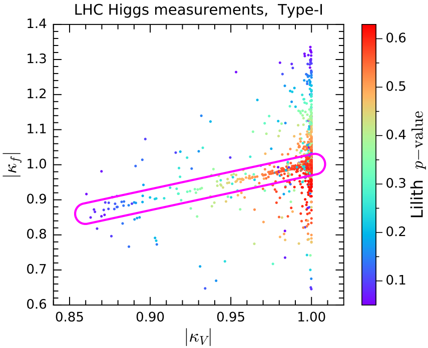
- We utilize a numerical tool **Lilith** to construct an approximate likelihood based on experimental results of **Higgs signal strength measurements**
- Calculate the **likelihood** $-2\ln L$ for each parameter points based on Tevatron data as well as LHC Run 1 and Run 2 data from ATLAS and CMS
- Transform $-2\ln L$ to p-**value**, and select parameter points with $p > 0.05$, *i.e.*, discard parameter points that are excluded by data at 95% C.L.

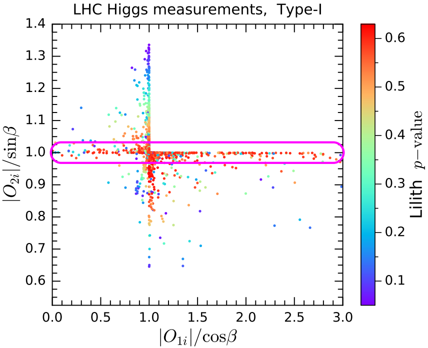


[ATLAS-CONF-2015-044/CMS-PAS-HIG-15-002; CMS coll., 1809.10733, EPJC]


LHC Higgs measurements, Type-I
$\lambda_{\rm SM} \simeq 0.26 \rightarrow$
λ_1
$\tan\beta$
Lilith p-value
$\tan\beta \ll 1$
$v_1 \gg v_2$
$\Phi_1 \simeq \Phi_h$
LHC Higgs measurements, Type-I
λ_2
$\tan\beta$
Lilith p-value
$\tan\beta \gg 1$
$v_2 \gg v_1$
$\Phi_2 \simeq \Phi_h$
LHC Higgs measurements, Type-I
$h_{\rm SM} = h_1 \rightarrow$
m_{h_1} (GeV)
$m_{h_{\rm SM}}$ (GeV)
Lilith p-value
LHC Higgs measurements, Type-I
$h_{\rm SM} = h_2$
$h_{\rm SM} = h_3$
m_{h_3} (GeV)
m_{h_2} (GeV)
Lilith p-value


LHC Higgs measurements, Type-I
$|\kappa_f|$
$|\kappa_V|$
0.85
0.90
0.95
1.00
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4
Lilith $p-$value
0.1
0.2
0.3
0.4
0.5
0.6
LHC Higgs measurements, Type-I
$|O_{2i}|/\sin\beta$
$|O_{1i}|/\cos\beta$
0.0
0.5
1.0
1.5
2.0
2.5
3.0
Lilith $p-$value



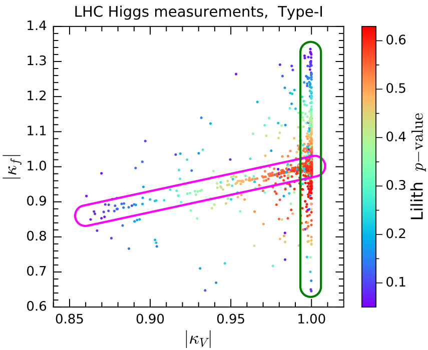


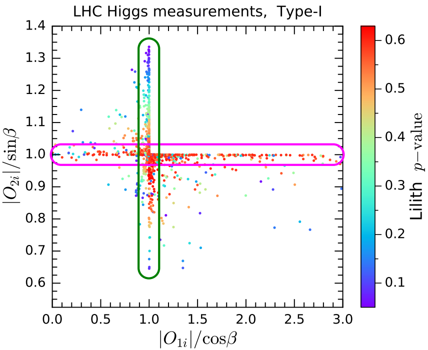
Category 1: $\kappa_V \simeq \kappa_f$ (nearly total positive correlation)

$\tan\beta \gg 1$ 👉 $\beta \simeq \pi/2$ 👉 $c_\beta O_{1i} + s_\beta O_{2i} = \kappa_V \simeq O_{2i} \simeq \kappa_f = O_{2i}/s_\beta$

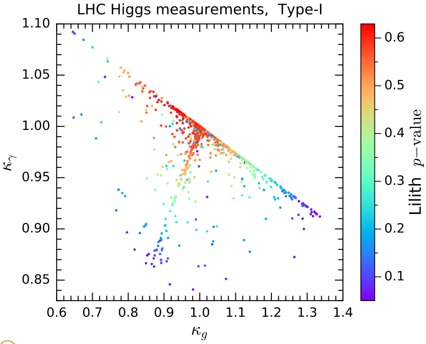
$|O_{2i}| \leq 1$ 👉 $|\kappa_V|, |\kappa_f| \leq 1$

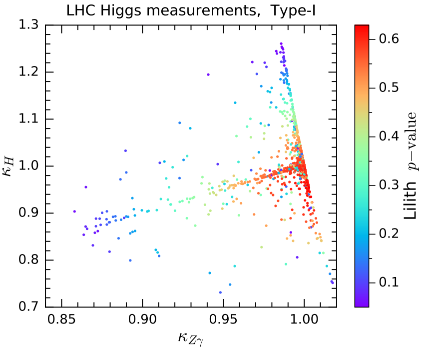
Most of parameter points in Category 1 correspond to $|O_{2i}|/s_\beta \simeq 1$





- **Category 1:** $\kappa_V \simeq \kappa_f$ (nearly total positive correlation)
  $\tan\beta \gg 1$ 👉 $\beta \simeq \pi/2$ 👉 $c_\beta O_{1i} + s_\beta O_{2i} = \kappa_V \simeq O_{2i} \simeq \kappa_f = O_{2i}/s_\beta$
  $|O_{2i}| \le 1$ 👉 $|\kappa_V|, |\kappa_f| \le 1$
  Most of parameter points in Category 1 correspond to $|O_{2i}|/s_\beta \simeq 1$
- **Category 2:** $|\kappa_V| \simeq 1$ with varying $|\kappa_f|$, corresponding to $|O_{1i}|/c_\beta \simeq 1$
  $|O_{1i}| \simeq c_\beta$, $|O_{2i}| \simeq s_\beta$ 👉 $|\kappa_V| = |c_\beta O_{1i} + s_\beta O_{2i}| \simeq c_\beta^2 + s_\beta^2 = 1$
  For small β, the 2nd relation $|O_{2i}| \simeq s_\beta$ is not important for $|\kappa_V| \simeq 1$



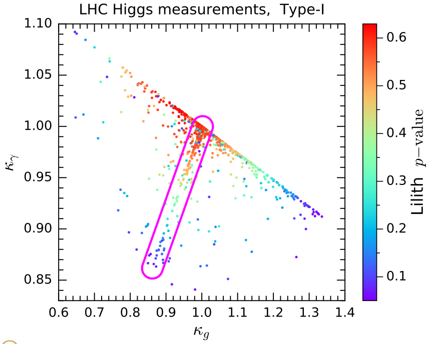


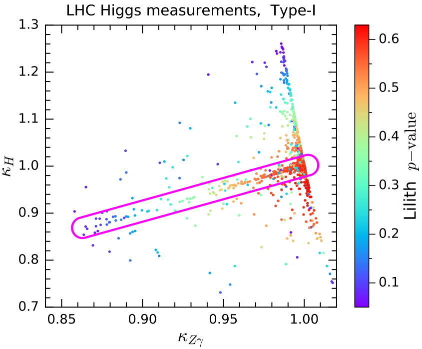
Parametrizations for κ_g, κ_γ, $\kappa_{Z\gamma}$, and κ_H [PDG 2018]

$$\kappa_g^2 = 1.06\kappa_t^2 + 0.01\kappa_b^2 - 0.07\kappa_t\kappa_b$$

$$\kappa_\gamma^2 = 1.59\kappa_W^2 + 0.07\kappa_t^2 - 0.66\kappa_W\kappa_t, \quad \kappa_{Z\gamma}^2 = 1.12\kappa_W^2 + 0.03\kappa_t^2 - 0.15\kappa_W\kappa_t$$

$$\kappa_H^2 = 0.57\kappa_b^2 + 0.06\kappa_\tau^2 + 0.03\kappa_c^2 + 0.22\kappa_W^2 + 0.03\kappa_Z^2 + 0.09\kappa_g^2 + 0.0023\kappa_\gamma^2$$





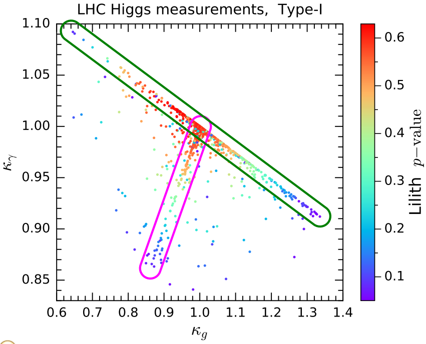
Parametrizations for κ_g, κ_γ, $\kappa_{Z\gamma}$, and κ_H [PDG 2018]

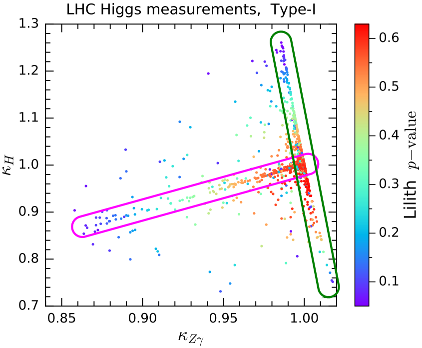
$$\kappa_g^2 = 1.06\kappa_t^2 + 0.01\kappa_b^2 - 0.07\kappa_t\kappa_b$$

$$\kappa_\gamma^2 = 1.59\kappa_W^2 + 0.07\kappa_t^2 - 0.66\kappa_W\kappa_t, \quad \kappa_{Z\gamma}^2 = 1.12\kappa_W^2 + 0.03\kappa_t^2 - 0.15\kappa_W\kappa_t$$

$$\kappa_H^2 = 0.57\kappa_b^2 + 0.06\kappa_\tau^2 + 0.03\kappa_c^2 + 0.22\kappa_W^2 + 0.03\kappa_Z^2 + 0.09\kappa_g^2 + 0.0023\kappa_\gamma^2$$

Category 1: $\kappa_V \simeq \kappa_f$ κ_g ($\kappa_{Z\gamma}$) is **positively** correlated to κ_γ (κ_H)





Parametrizations for κ_g, κ_γ, $\kappa_{Z\gamma}$, and κ_H [PDG 2018]

$$\kappa_g^2 = 1.06\kappa_t^2 + 0.01\kappa_b^2 - 0.07\kappa_t\kappa_b$$

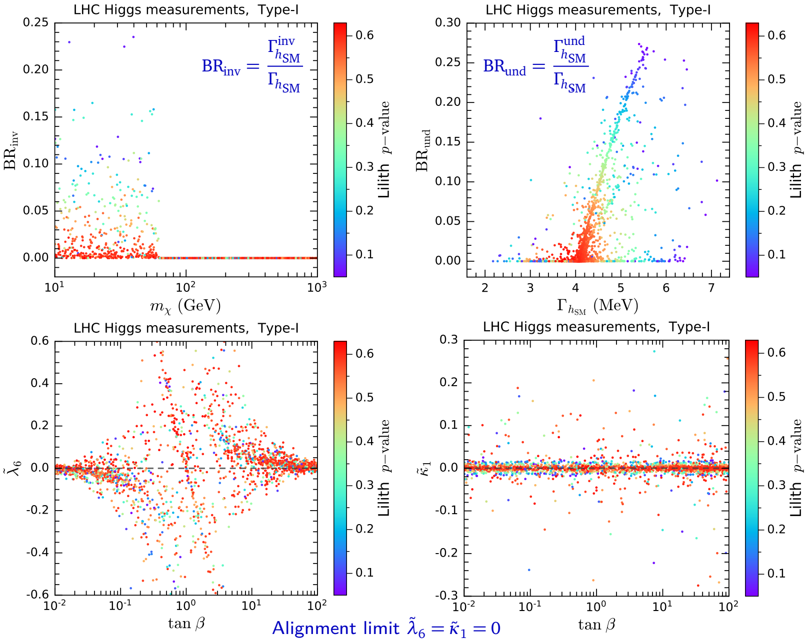
$$\kappa_\gamma^2 = 1.59\kappa_W^2 + 0.07\kappa_t^2 - 0.66\kappa_W\kappa_t, \quad \kappa_{Z\gamma}^2 = 1.12\kappa_W^2 + 0.03\kappa_t^2 - 0.15\kappa_W\kappa_t$$

$$\kappa_H^2 = 0.57\kappa_b^2 + 0.06\kappa_\tau^2 + 0.03\kappa_c^2 + 0.22\kappa_W^2 + 0.03\kappa_Z^2 + 0.09\kappa_g^2 + 0.0023\kappa_\gamma^2$$

- **Category 1:** $\kappa_V \simeq \kappa_f$ ☞ κ_g ($\kappa_{Z\gamma}$) is **positively** correlated to κ_γ (κ_H)
- **Category 2:** $|\kappa_V| \simeq 1$ with varying $|\kappa_f|$

  $\kappa_V\kappa_f > 0$ satisfied, κ_g (κ_γ) is **positively** (**negatively**) correlated to $|\kappa_f|$

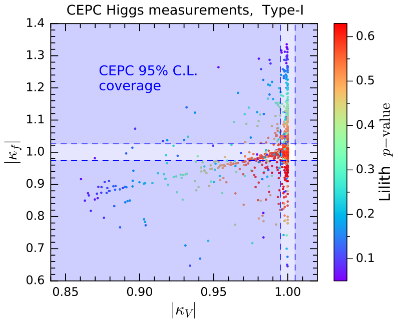
  κ_H ($\kappa_{Z\gamma}$) is **positively** (**negatively**) correlated to $|\kappa_f|$

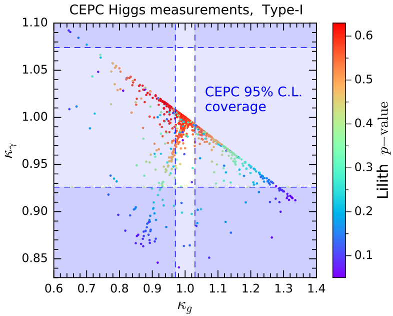


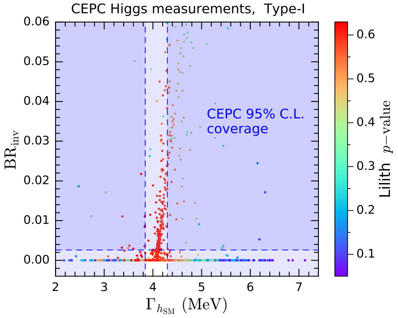
Alignment limit $\tilde{\lambda}_6 = \tilde{\kappa}_1 = 0$

Future CEPC Higgs measurements

- Relative 1σ precisions: $\delta\kappa_b = 1.3\%$, $\delta\kappa_Z = 0.25\%$, $\delta\kappa_g = 1.5\%$, $\delta\kappa_\gamma = 3.7\%$, $\delta\Gamma_{h_{\mathrm{SM}}}/\Gamma_{h_{\mathrm{SM}}} = 2.8\%$ [CEPC CDR, 1811.10545]
- 95% C.L. upper limit on invisible Higgs decays: $\mathrm{BR_{inv}} < 0.26\%$ [Tan et al., 2001.05912, CPC]





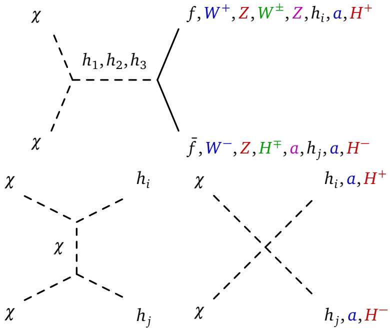


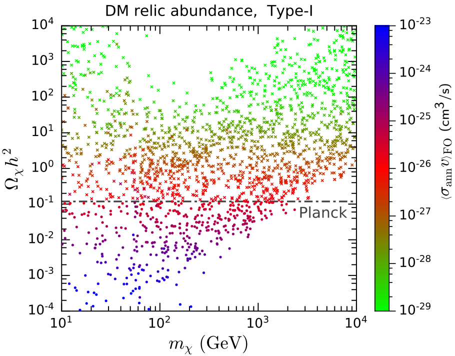
DM Relic Abundance

- **Planck** observed DM relic abudance $\Omega_{\rm DM}h^2 = 0.1186 \pm 0.0020$

[Planck coll., 1502.01589, Astron. Astrophys.]

- Numerical tools: FeynRules 👉 MadGraph 👉 MadDM 👉 $\Omega_\chi h^2$





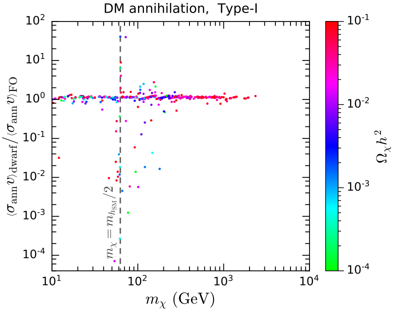
- **Colored dots:** $\Omega_\chi h^2$ is **equal** or **lower** than observation
- **Colored crosses:** χ is **overproduced**, contradicting standard cosmology
- For $m_\chi \gtrsim 3$ TeV, the observed relic abundance **could not** be achieved

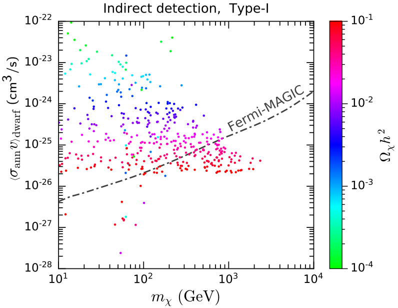
Indirect Detection

✨ **Dwarf galaxies** are the **largest substructures** of the **Galactic dark halo**

👉 Perfect targets for **γ-ray indirect detection** experiments

🔧 We utilize MadDM to calculate $\langle\sigma_{\rm ann} v\rangle_{\rm dwarf}$ with a typical average velocity of DM particles in dwarf galaxies $v_0 = 2 \times 10^{-5}$





🌗 $\langle\sigma_{\rm ann} v\rangle_{\rm dwarf}$ differs from the freeze-out value $\langle\sigma_{\rm ann} v\rangle_{\rm FO}$ due to **resonance effect**

🌓 The parameter points with $m_\chi \gtrsim 100$ GeV and $\Omega_\chi h^2 \sim 0.1$ are **not excluded** by Fermi-LAT and MAGIC γ-ray observations [MAGIC & Fermi-LAT, 1601.06590, JCAP]

Effective Potential

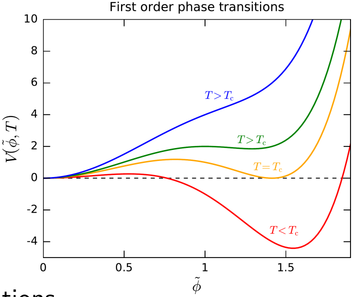
Different **local minima** in the **effective potential** V_{eff} of the scalar fields

Different **phases** Phase transitions

We assume that only the CP-even neutral scalar fields (ρ_1, ρ_2, s) develop VEVs in the cosmological history

As a function of the **classical background fields** $(\tilde{\rho}_1, \tilde{\rho}_2, \tilde{s})$ and the **temperature** T,

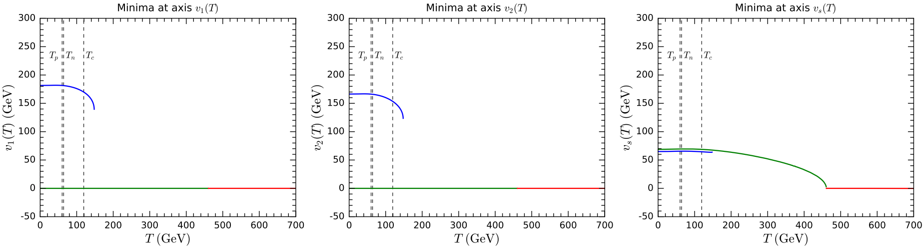
$$V_{\text{eff}}(\tilde{\rho}_1, \tilde{\rho}_2, \tilde{s}, T) = V_0 + V_1 + V_{\text{CT}} + V_{1\text{T}} + V_{\text{D}}$$



- Tree-level potential V_0
- 1-loop zero-temperature corrections V_1
- Counter terms V_{CT} for keeping the VEV positions and the renormalized mass-squared matrix of the CP-even neutral scalars
- 1-loop finite-temperature corrections $V_{1\text{T}}(T)$
- Daisy diagram contributions $V_{\text{D}}(T)$ beyond 1-loop at finite temperature

Temperature Evolution of Local Minima

- We utilize CosmoTransitions to analyze the phase transitions
- At sufficiently high temperatures, the only minimum in the effective potential is the **gauge symmetric minimum** $(\tilde{\rho}_1, \tilde{\rho}_2, \tilde{s}) = (0, 0, 0)$
- As the Universe cools down, **extra minima** may appear
- **Multi-step** cosmological phase transitions typically occur in this model
- If there are two coexisted minima separated by a **high barrier**, a **strong FOPT** could take place, resulting in **stochastic gravitational waves**
- At last, the system is trapped at the **true vacuum** $(\tilde{\rho}_1, \tilde{\rho}_2, \tilde{s}) = (v_1, v_2, v_s)$



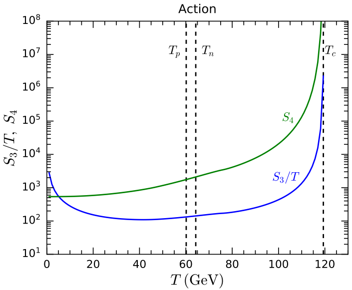
Bubble Nucleation in a FOPT

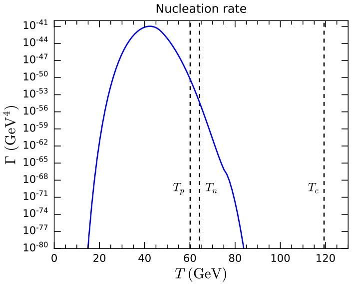
- A FOPT from a **false vacuum** to the **true vacuum** nucleates **bubbles**, inside which the system is trapped at the true vacuum
- **Bubble nucleation rate** $\Gamma \sim T^4 e^{-S}$
- The action $S = \min(S_4, S_3/T)$
- O(4)-symmetric quantum tunneling action S_4
- O(3)-symmetric thermal fluctuation action

$$S_3 = 4\pi \int_0^\infty dr\, r^2 \left[\frac{1}{2}\frac{d\phi_i}{dr}\frac{d\phi_i}{dr} + V_{\text{eff}}(\phi_i, T)\right]$$

- Bounce solution $\phi_i(r) = \big(\tilde{\rho}_1(r), \tilde{\rho}_2(r), \tilde{s}(r)\big)$ with the bubble radius r satisfying

$$\begin{cases} \dfrac{d^2\phi_i}{dr^2} + \dfrac{2}{r}\dfrac{d\phi_i}{dr} = \dfrac{\partial V_{\text{eff}}}{\partial \phi_i} \\ \left.\dfrac{d\phi_i}{dr}\right|_{r=0} = 0, \quad \phi_i(\infty) = \phi_i^{\text{false}} \end{cases}$$





Key Quantities of a FOPT

☀️ The **released vacuum energy density** in the FOPT

$$\rho_{\text{vac}} = V_{\text{eff}}(\phi_i^{\text{false}}, T) - V_{\text{eff}}(\phi_i^{\text{true}}, T) - T\frac{\partial}{\partial T}[V_{\text{eff}}(\phi_i^{\text{false}}, T) - V_{\text{eff}}(\phi_i^{\text{true}}, T)]$$

👉 **Gradient energy** of the scalar field $\Rightarrow$ **Bubble expansion** 🎈

👉 **Thermal energy** 🔥 and **bulk kinetic energy** 🚿 of the plasma

💪 **Phase transition strength** $\alpha \equiv \dfrac{\rho_{\text{vac}}}{\rho_{\text{rad}}}$, where $\rho_{\text{rad}} = \dfrac{\pi^2}{30} g_* T^4$ is the radiation energy density in the plasma with g_* the effective relativistic degrees of freedom

⌛ $\beta(T') \equiv -\left.\dfrac{dS}{dt}\right|_{t=t'} = \left.\left(HT\dfrac{dS}{dT}\right)\right|_{T=T'}$ roughly describes the **inverse time duration** of the FOPT at a characteristic temperature T'

👉 A larger α implies a **stronger** FOPT, and a smaller β means a **longer** FOPT

✏️ The dimensionless quantity $\tilde{\beta}(T') \equiv \dfrac{\beta(T')}{H(T')}$ compares the cosmological expansion time scale H^{-1} with the phase transition time scale β^{-1} at $T = T'$

Key Temperatures of a FOPT

🌙 **Critical temperature** T_c : the potential values at the two minima are equal

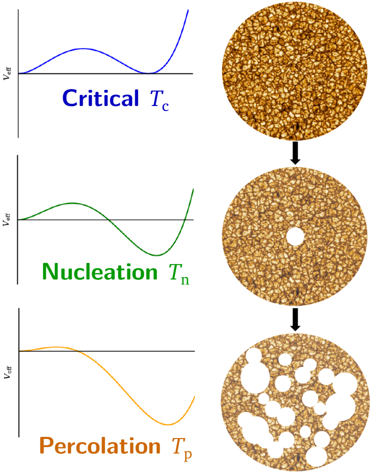
⭐ **Nucleation temperature** T_n : one single bubble is nucleated within a Hubble volume

$$\frac{S_3(T_n)}{T_n} \simeq 141.5 - 2\ln\frac{g_*}{100} - 4\ln\frac{T_n}{100\ \text{GeV}} - \ln\frac{\tilde{\beta}(T_n)}{100}$$

✨ **Percolation temperature** T_p : percolation occurs when the fraction of space converted to the true vacuum reaches $\sim 29\%$, corresponding to the **maximum of bubble collisions**

$$\frac{S_3(T_p)}{T_p} \simeq 132.0 - 2\ln\frac{g_*}{100} - 4\ln\frac{T_p}{100\ \text{GeV}} - 4\ln\frac{\tilde{\beta}(T_p)}{100} + 3\ln v_w$$

✏️ v_w is the **velocity of the bubble wall**



[Wang, Huang, Zhang, 2003.08892, JCAP]

Bubble Expansion in the Plasma

The bubble expansion depends on the interactions between the bubble wall and the plasma, analogous to **chemical combustion in a relativistic fluid**

Hydrodynamic analyses show that bubble expansion have various **modes**

Subsonic deflagrations Supersonic deflagrations (hybrid)

Jouguet detonations Weak detonations Runway bubble walls

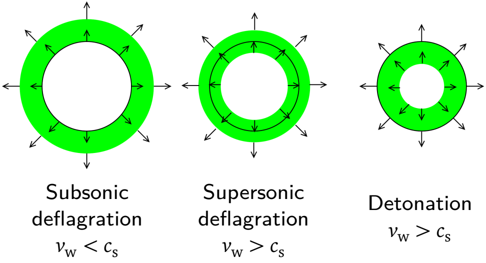
It is difficult to completely work out the bubble wall velocity v_w

For **Jouguet detonations**, the Chapman-Jouguet condition leads to a bubble wall velocity of

$$v_\mathrm{CJ} = \frac{1+\sqrt{3\alpha^2+2\alpha}}{\sqrt{3}(1+\alpha)},$$

which is larger than the **sound speed** in the plasma $c_\mathrm{s} \simeq 1/\sqrt{3}$

This is a **typical** assumption when evaluating GW signals



[Espinosa et al., 1004.4187, JCAP]

Energy Budget of a FOPT

Define **efficiency factors** by the fractions of the available vacuum energy

κ_ϕ : the fraction converted into the **gradient energy** of the scalar fields

- It is typically **negligible**, except for runaway bubble walls ($v_w \to 1$)

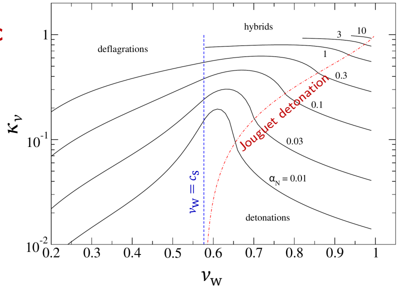
κ_v : the fraction converted into the kinetic energy of the **fluid bulk motion**

- It depends on the **FOPT strength** α and the **bubble wall velocity** v_w

κ_{turb} : the fraction converted into the kinetic energy of **magnetohydrodynamic (MHD) turbulence**

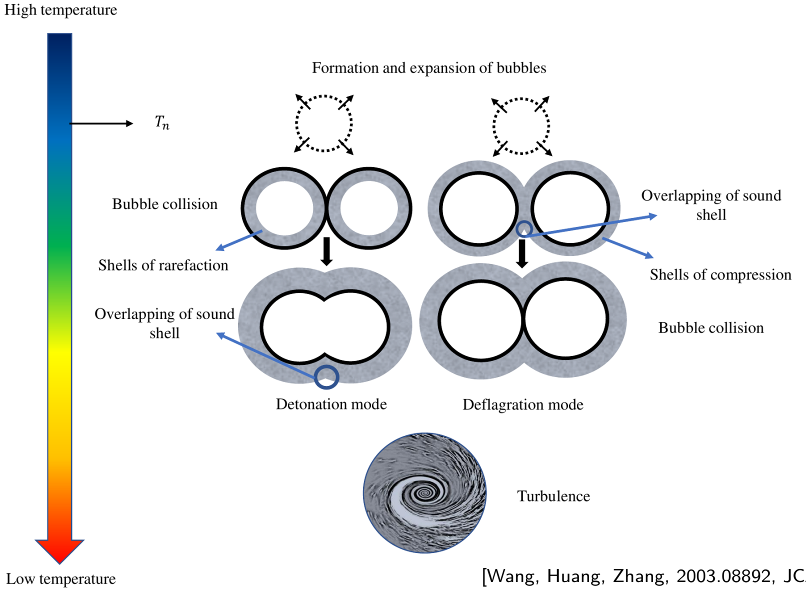
- Recent simulations suggest that $\kappa_{\text{turb}} \simeq 5\text{–}10\%\,\kappa_v$ at most

For **Jouguet detonations**, $v_w = v_{CJ}$, and $\kappa_v^{CJ} = \dfrac{\sqrt{\alpha}}{0.135 + \sqrt{0.98 + \alpha}}$



[Espinosa et al., 1004.4187, JCAP]

Physical Processes in a FOPT



[Wang, Huang, Zhang, 2003.08892, JCAP]

GW Sources in a FOPT

An electroweak FOPT could induce significant **perturbations** of the metric and produce **stochastic GWs around $f \sim$ mHz**, whose spectrum depend on α and $\tilde{\beta}$ at $t = t_*$ (corresponding to $T \sim T_\mathrm{p}$) when the GWs are produced

The resulting **GW spectrum** is commonly expressed as $\Omega_\mathrm{GW} = \dfrac{f}{\rho_\mathrm{c}} \dfrac{d\rho_\mathrm{GW}}{df}$

ρ_GW is the present GW energy density, ρ_c is the critical density

Three **GW sources**: $\Omega_\mathrm{GW} = \Omega_\mathrm{col} + \Omega_\mathrm{sw} + \Omega_\mathrm{turb}$

Bubble collisions: $\Omega_\mathrm{col} \propto \kappa_\phi^2$ is **negligible** except for runaway bubble walls

Sound waves: sound shells propagate into the fluid as sound waves

$$\Omega_\mathrm{sw} h^2 = 1.17 \times 10^{-6} \, \frac{\Upsilon v_\mathrm{w}}{\tilde{\beta}} \left(\frac{\kappa_v \alpha}{1+\alpha}\right)^2 \left(\frac{100}{g_*}\right)^{1/3} \left(\frac{f}{f_\mathrm{sw}}\right)^3 \left(\frac{7}{4 + 3f^2/f_\mathrm{sw}^2}\right)^{7/2}$$

This is the **dominant** source; Υ accounts for the duration of sound waves

MHD turbulence: bubble collisions stir up turbulence in the fluid

$$\Omega_\mathrm{turb} h^2 = 3.35 \times 10^{-4} \, \frac{v_\mathrm{w}}{\tilde{\beta}} \left(\frac{\kappa_\mathrm{turb} \alpha}{1+\alpha}\right)^{3/2} \left(\frac{100}{g_*}\right)^{1/3} \frac{(f/f_\mathrm{turb})^3}{(1 + f/f_\mathrm{turb})^{11/3} (1 + 8\pi f/h_*)}$$

GW Signals from pNGB DM and 2 Higgs Doublets

Random scans for **Type-I** and **Type-II** Yukawa couplings

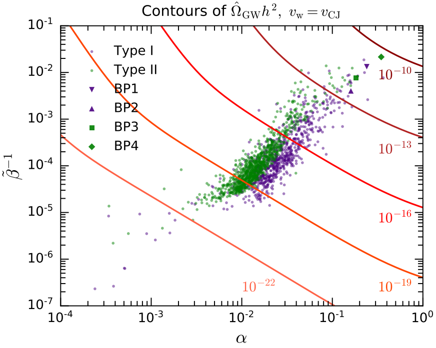
$$10\ \text{GeV} < v_s < 1\ \text{TeV}, \quad 58\ \text{GeV} < m_\chi < 800\ \text{GeV},$$
$$\text{GeV}^2 < |m_{12}^2| < (500\ \text{GeV})^2, \quad 0.5 < \tan\beta < 20,$$
$$0.8 < \lambda_1, \lambda_2, \lambda_S, |\lambda_3|, |\lambda_4|, |\lambda_5| < 8, \quad 0.01 < |\kappa_1|, |\kappa_2| < 8$$

The parameter points are required to give an observed DM relic abundance, and to pass all the existed experimental constraints, and to cause a **FOPT**

The resulting **relic GW spectra** are further estimated, assuming **Jouguet detonations** with $v_w = v_{CJ}$

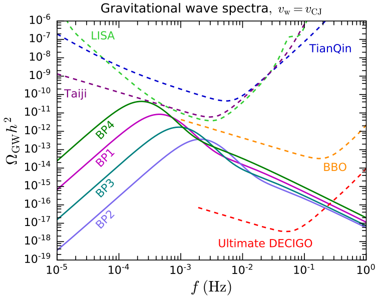
Contours correspond to the **peak amplitude** of the GW spectrum $\hat{\Omega}_{GW}h^2$

A **larger** α and a **larger** $\tilde{\beta}^{-1}$ imply a stronger and longer FOPT, leading to a **more significant** GW signal



Benchmark Points (BPs)

	BP1	BP2	BP3	BP4
Type	I	I	II	II
v_s (GeV)	542.40	384.26	64.987	138.82
m_χ (GeV)	117.88	78.191	134.03	76.678
m_{12}^2 (10^4 GeV^2)	2.0210	0.015876	17.696	15.042
$\tan\beta$	2.8616	3.2654	0.91655	1.1732
λ_1	2.1496	2.1882	1.5297	0.87839
λ_2	0.80887	0.85479	1.2074	0.80222
λ_3	2.3925	2.2628	1.5741	2.8002
λ_4	3.0027	1.4715	5.3967	4.4643
λ_5	−6.2187	−4.0567	−7.8556	−7.5755
λ_S	3.4048	2.5502	6.0689	4.8644
κ_1	−1.4852	1.0295	0.80378	−0.38075
κ_2	1.1727	−1.2142	−0.83745	−0.14591
m_{h_1} (GeV)	125.11	91.459	125.38	124.87
m_{h_2} (GeV)	282.02	124.77	158.83	307.56
m_{h_3} (GeV)	1014.5	641.83	650.98	582.08
m_a (GeV)	664.75	496.49	911.87	874.04
$m_{H^\pm}$ (GeV)	402.96	280.94	655.60	631.66
$\langle\sigma_{\rm ann}v\rangle_{\rm dwarf}$ (10^{-26} cm^3/s)	1.30	0.368	1.72	0.682
α	**0.240**	**0.160**	**0.181**	**0.346**
$\tilde\beta^{-1}$ (10^{-2})	**1.33**	**0.402**	**0.771**	**2.15**
$T_{\rm p}$ (GeV)	55.3	74.9	60.2	47.2
$\rm SNR_{LISA}$	96.6	37.7	60.1	120
$\rm SNR_{Taiji}$	83.3	23.9	42.3	155
$\rm SNR_{TianQin}$	5.50	2.39	3.07	9.20



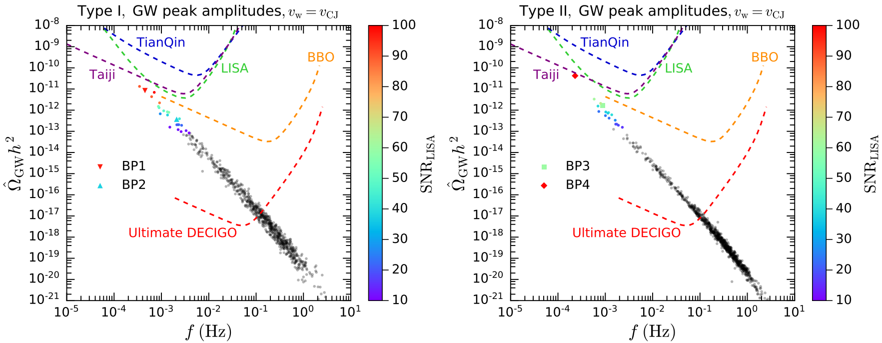
For a practical observation time $\mathcal{T}$, the **signal-to-noise ratio** is

$$\mathrm{SNR} \equiv \sqrt{\mathcal{T}\int_{f_{\min}}^{f_{\max}} \frac{\Omega_{\rm GW}^2(f)}{\Omega_{\rm sens}^2(f)}\, df}$$

Take $\mathcal{T} = 3$ yr for LISA, Taiji, TianQin

For the six (four) link configuration, the detection threshold is $\mathrm{SNR_{thr}} = 10$ (50)

Peak Amplitudes and Signal-to-noise Ratios



🎨 The **colored points** leads to $\mathrm{SNR}_{\mathrm{LISA}} > 10$, promising to be probed by **LISA**

✨ Based on current information, the sensitivity of **Taiji** could be similar to LISA, while **TianQin** may be somehow less sensitive

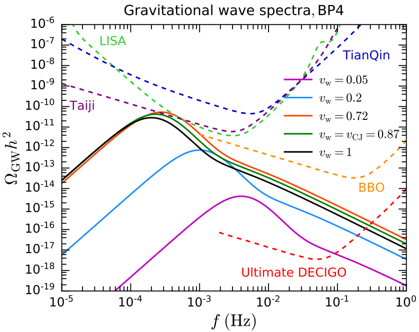
🎯 Far future plans aiming at $f \sim \mathcal{O}(0.1)$ Hz, like **BBO** and **DECIGO**, may explore much more parameter points

Dependence on Bubble Expansion Modes and v_w

The previous results for GW signals are estimated by assuming **Jouguet detonations** with $v_w = v_{CJ}$

For different **bubble expansion modes**, the dependence of κ_v on v_w and α is different

In order to show such a dependence, we additionally estimate the GW spectra for **BP4** under the following assumptions



- **Subsonic deflagrations** with $v_w = 0.05$ ☞ very weak GW signal
- **Subsonic deflagrations** with $v_w = 0.2$ ☞ weak GW signal
- **Detonations** with $v_w = 1$ ☞ strong GW signal
- **Jouguet detonations** with $v_w = v_{CJ} = 0.87$ ☞ $\mathrm{SNR}_{\mathrm{TianQin}} = 9.2$
- **Supersonic deflagrations** with $v_w = 0.72$ ☞ strongest GW signal

☞ $\mathrm{SNR}_{\mathrm{TianQin}} = 15.8$ ☞ could be properly tested by TianQin

Summary

- We study the **pNGB DM framework** with two Higgs doublets
- Because of the pNGB nature of the DM candidate χ, the tree-level **DM-nucleon scattering amplitude vanishes** in direct detection
- We perform a random scan to find the parameter points consistent with **current Higgs measurements**
- Some parameter points with 100 GeV $\lesssim m_\chi \lesssim 3$ TeV can give an **observed relic abundance** and evade the constraints from **indirect detection**
- We investigate the **electroweak FOPT** and the resulting **stochastic GWs**
- Some parameter points could induce strong GW signals, which have the opportunity to be probed in future **LISA**, **Taiji**, and **TianQin** experiments.

Summary

- We study the **pNGB DM framework** with two Higgs doublets
- Because of the pNGB nature of the DM candidate χ, the tree-level **DM-nucleon scattering amplitude vanishes** in direct detection
- We perform a random scan to find the parameter points consistent with **current Higgs measurements**
- Some parameter points with 100 GeV $\lesssim m_\chi \lesssim 3$ TeV can give an **observed relic abundance** and evade the constraints from **indirect detection**
- We investigate the **electroweak FOPT** and the resulting **stochastic GWs**
- Some parameter points could induce strong GW signals, which have the opportunity to be probed in future **LISA**, **Taiji**, and **TianQin** experiments.

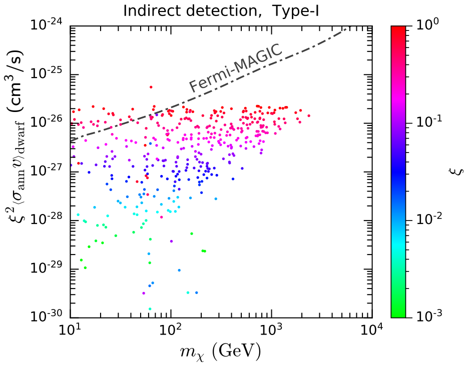
Thanks for your attention!

Rescaling with a Fraction ξ

✨ Assume the relic abundance of χ is solely determined by thermal mechanism

👉 χ could just constitute a **fraction** of all dark matter, $\xi = \dfrac{\Omega_\chi}{\Omega_{\rm DM}}$

👉 $\chi\chi$ annihilation cross section in dwarf galaxies should be effectively rescaled to $\xi^2 \langle\sigma_{\rm ann} v\rangle_{\rm dwarf}$ for comparing with the Fermi-MAGIC constraint



Constraints from Flavor Physics and DM Indirect Detection