CEPC Precision of Electroweak Oblique Parameters and Weakly Interacting Dark Matter

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Based on **Chengfeng Cai, ZHY, Hong-Hao Zhang, arXiv:1611.02186**

CoEPP 2017 Annual Workshop

23 February 2017, Adelaide

Dark Matter

Dark matter (DM) makes up most of the matter component in the Universe, as suggested by astrophysical and cosmological observations

If DM particles (χ) were thermally produced in the early Universe, their **relic abundance** would be determined by the annihilation cross section $\langle \sigma_{\text{ann}} v \rangle$:

$$\Omega_\chi h^2 \simeq \frac{3 \times 10^{-27}\ \text{cm}^3/\text{s}}{\langle \sigma_{\text{ann}} v \rangle}$$

$$\text{Observation value } \Omega_\chi h^2 \simeq 0.1 \quad \Rightarrow \quad \langle \sigma_{\text{ann}} v \rangle \simeq 3 \times 10^{-26}\ \text{cm}^3/\text{s}$$

Assuming the annihilation process consists of two weak interaction vertices with the $\mathrm{SU(2)_L}$ gauge coupling $g \simeq 0.64$, for $m_\chi \sim \mathcal{O}(\text{TeV})$ we have

$$\langle \sigma_{\text{ann}} v \rangle \sim \frac{g^4}{16\pi^2 m_\chi^2} \sim \mathcal{O}(10^{-26})\ \text{cm}^3/\text{s}$$

$\Rightarrow$ A very attractive class of DM candidates:

Weakly interacting massive particles (WIMPs)

CEPC Project

The **Circular Electron Positron Collider (CEPC)**, proposed by the Chinese HEP community, will mainly serve as a Higgs factory at $\sqrt{s} \sim 240$ GeV

The **preliminary conceptual design report** was released in May 2015: `http://cepc.ihep.ac.cn/preCDR/volume.html`

Its low-energy plans will operate at the Z pole ($\sqrt{s} \sim 91$ GeV, 10^{10} Z bosons) and near the WW threshold ($\sqrt{s} \sim 160$ GeV), leading to great improvements for **electroweak (EW) precision measurements**

WIMP models typically contain colorless **EW multiplets** whose electrically neutral components serve as DM candidates; such multiplets will affect EW precision observables (or **oblique parameters**) via **loop corrections**

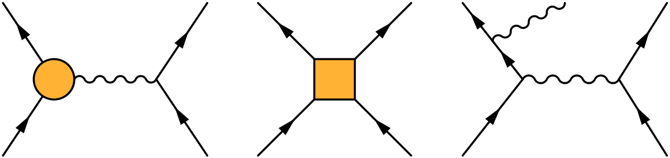
$$\Downarrow$$

CEPC provides an excellent opportunity to indirectly probe WIMP DM models

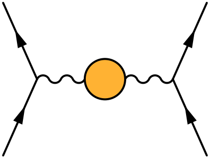
Electroweak Radiative Corrections

Two classes of EW radiative corrections

- **Direct Corrections:** vertex, box, and bremsstrahlung corrections



- **Oblique Corrections:** gauge boson propagator corrections



Oblique corrections can be treated in a self-consistent and model-independent way through an effective lagrangian to incorporate a large class of Feynman diagrams into a few **running couplings** [Kennedy & Lynn, NPB 322, 1 (1989)]

Electroweak Oblique Parameters

EW oblique parameters S, T, and U are further introduced to describe **new physics contributions** through oblique corrections [Peskin & Takeuchi, '90, '92]

$$S = 16\pi[\Pi'_{33}(0) - \Pi'_{3Q}(0)]$$

$$T = \frac{4\pi}{s_W^2 c_W^2 m_Z^2}[\Pi_{11}(0) - \Pi_{33}(0)], \quad U = 16\pi[\Pi'_{11}(0) - \Pi'_{33}(0)]$$

Here $\Pi'_{IJ}(0) \equiv \partial \Pi_{IJ}(p^2)/\partial p^2|_{p^2=0}$, $s_W \equiv \sin\theta_W$, $c_W \equiv \cos\theta_W$

γ  γ $= ie^2\Pi_{QQ}(p^2)g^{\mu\nu} + (p^\mu p^\nu \text{ terms})$

Z  γ $= \dfrac{ie^2}{s_W c_W}[\Pi_{3Q}(p^2) - s_W^2\Pi_{QQ}(p^2)]g^{\mu\nu} + (p^\mu p^\nu \text{ terms})$

Z  Z $= \dfrac{ie^2}{s_W^2 c_W^2}[\Pi_{33}(p^2) - 2s_W^2\Pi_{3Q}(p^2) + s_W^4\Pi_{QQ}(p^2)]g^{\mu\nu} + (p^\mu p^\nu \text{ terms})$

W  W $= \dfrac{ie^2}{s_W^2}\Pi_{11}(p^2)g^{\mu\nu} + (p^\mu p^\nu \text{ terms})$

Custodial Symmetry

Standard model (SM) scalar potential $V = -\mu^2 H^\dagger H + \lambda(H^\dagger H)^2$ is a function of $H^\dagger H$, which respects an $\mathrm{SU}(2)_\mathrm{L} \times \mathrm{SU}(2)_\mathrm{R}$ **global symmetry**:

$$H^\dagger H = -\frac{1}{2}\epsilon_{AB}\epsilon^{ij}(\mathcal{H}^A)_i(\mathcal{H}^B)_j, \quad (\mathcal{H}^A)_i \equiv \begin{pmatrix} H_i^\dagger \\ H_i \end{pmatrix} \text{ is an } \mathrm{SU}(2)_\mathrm{R} \text{ doublet}$$

$$H \to \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix} \quad \Rightarrow \quad \mathrm{SU}(2)_\mathrm{L} \times \mathrm{SU}(2)_\mathrm{R} \to \mathrm{SU}(2)_\mathrm{L+R} \text{ **custodial symmetry**}$$

$\Downarrow$

$\mathrm{SU}(2)_\mathrm{L}$ gauge bosons W_μ^a transform as an $\mathrm{SU}(2)_\mathrm{L+R}$ triplet and acquire the same mass from EW symmetry breaking

$\Downarrow$

The custodial symmetry protects the tree-level relation $\rho \equiv m_W^2/(m_Z^2 c_\mathrm{W}^2) = 1$ up to EW radiative corrections [Sikivie *et al.*, NPB 173, 189 (1980)], and leads to $T = U = 0$ (note that $\rho - 1 = \alpha T$)

The custodial symmetry is **approximate** in the SM, explicitly broken by the Yukawa couplings of fermions and the $\mathrm{U}(1)_\mathrm{Y}$ gauge interaction

Electroweak Precision Observables

For evaluating CEPC precision of oblique parameters, we use a simplified set of EW precision observables in the **global fit**:

$$\alpha_s(m_Z^2),\ \Delta\alpha_{\rm had}^{(5)}(m_Z^2),\ m_Z,\ m_t,\ m_h,\ m_W,\ \sin^2\theta_{\rm eff}^\ell,\ \Gamma_Z$$

Free parameters: the former 5 observables, S, T, and U

The remaining 3 observables are determined by the free parameters:

$$m_W = m_W^{\rm SM}\left[1-\frac{\alpha}{4(c_{\rm W}^2-s_{\rm W}^2)}(S-1.55T-1.24U)\right]$$

$$\sin^2\theta_{\rm eff}^\ell = (\sin^2\theta_{\rm eff}^\ell)^{\rm SM}+\frac{\alpha}{4(c_{\rm W}^2-s_{\rm W}^2)}(S-0.69T)$$

$$\Gamma_Z = \Gamma_Z^{\rm SM}-\frac{\alpha^2 m_Z}{72 s_{\rm W}^2 c_{\rm W}^2(c_{\rm W}^2-s_{\rm W}^2)}(12.2S-32.9T)$$

The calculation of **SM predictions** is based on 2-loop radiative corrections

CEPC Precision of Electroweak Observables

	Current data	CEPC-B precision	CEPC-I precision
$\alpha_s(m_Z^2)$	0.1185 ± 0.0006	$\pm 1 \times 10^{-4}$	
$\Delta\alpha_{\rm had}^{(5)}(m_Z^2)$	0.02765 ± 0.00008	$\pm 4.7 \times 10^{-5}$	
m_Z [GeV]	91.1875 ± 0.0021	$\pm 5 \times 10^{-4}$	$\pm 1 \times 10^{-4}$
m_t [GeV]	$173.34 \pm 0.76_{\rm ex} \pm 0.5_{\rm th}$	$\pm 0.2_{\rm ex} \pm 0.5_{\rm th}$	$\pm 0.03_{\rm ex} \pm 0.1_{\rm th}$
m_h [GeV]	125.09 ± 0.24	$\pm 5.9 \times 10^{-3}$	
m_W [GeV]	$80.385 \pm 0.015_{\rm ex} \pm 0.004_{\rm th}$	$(\pm 3_{\rm ex} \pm 1_{\rm th}) \times 10^{-3}$	
$\sin^2\theta_{\rm eff}^{\ell}$	0.23153 ± 0.00016	$(\pm 2.3_{\rm ex} \pm 1.5_{\rm th}) \times 10^{-5}$	
Γ_Z [GeV]	2.4952 ± 0.0023	$(\pm 5_{\rm ex} \pm 0.8_{\rm th}) \times 10^{-4}$	$(\pm 1_{\rm ex} \pm 0.8_{\rm th}) \times 10^{-4}$

For **CEPC baseline (CEPC-B) precisions**, experimental uncertainties will be mostly reduced by CEPC measurements; theoretical uncertainties of m_W, $\sin^2\theta_{\rm eff}^{\ell}$, and Γ_Z can be reduced by fully calculating 3-loop corrections in the future

CEPC improved (CEPC-I) precisions need

- A high-precision beam energy calibration for improving m_Z and Γ_Z measurements
- A $t\bar{t}$ threshold scan for the m_t measurement at other e^+e^- colliders, like ILC

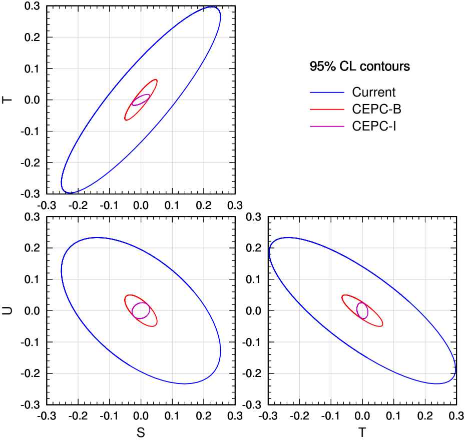
Global Fit

We use a **modified χ^2 function** [Fan, Reece & Wang, 1411.1054] for the global fit:

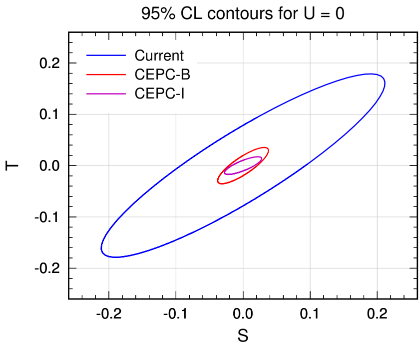
$$\sum_i \left(\frac{O_i^{\text{meas}} - O_i^{\text{pred}}}{\sigma_i} \right)^2 + \sum_j \left\{ -2 \ln \left[\operatorname{erf}\left(\frac{O_j^{\text{meas}} - O_j^{\text{pred}} + \delta_j}{\sqrt{2}\sigma_j} \right) - \operatorname{erf}\left(\frac{O_j^{\text{meas}} - O_j^{\text{pred}} - \delta_j}{\sqrt{2}\sigma_j} \right) \right] \right\}$$

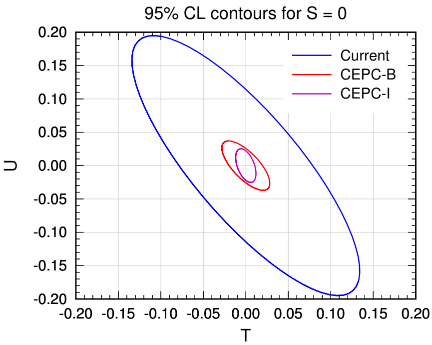
The **experimental uncertainty σ_j** and the **theoretical uncertainty δ_j** of an observable O_j are treated as **Gaussian** and **flat** errors

	Current	CEPC-B	CEPC-I
σ_S	0.10	0.021	0.011
σ_T	0.12	0.026	0.0071
σ_U	0.094	0.020	0.010
ρ_{ST}	+0.89	+0.90	+0.74
ρ_{SU}	−0.55	−0.68	+0.15
ρ_{TU}	−0.80	−0.84	−0.21



Fit Results for Some Parameters Fixed to 0





$T = U = 0$ fixed

	Current	CEPC-B	CEPC-I
σ_S	0.037	0.0085	0.0068

$S = U = 0$ fixed

	Current	CEPC-B	CEPC-I
σ_T	0.032	0.0079	0.0042

DM Models with Electroweak Multiplets

We study the CEPC sensitivity to WIMP models with a dark sector consisting of **EW multiplets**. By imposing a Z_2 symmetry, the DM candidate would be the lightest mass eigenstate of the neutral components.

1. EW oblique parameters S, T, and U respond to **EW symmetry breaking**
   - **Mass splittings** among the multiplet components induced by the nonzero Higgs VEV would break the EW symmetry
     - $\Rightarrow$ **Nonzero oblique parameters**
   - If the Higgs VEV just gives a **common mass shift** to every components in a multiplet, the effect can be absorbed into the gauge-invariant mass term
     - $\Rightarrow$ No EW symmetry breaking effect manifests
     - $\Rightarrow$ **Vanishing S, T, and U**
2. S relates to the $\mathrm{U}(1)_\mathrm{Y}$ gauge field
   - $\Rightarrow$ A multiplet with **zero hypercharge cannot contribute to S**
3. Multiplet couplings to the Higgs respect a **custodial symmetry**
   - $\Rightarrow$ **Vanishing T and U**

Fermionic and Scalar Multiplets

In other to have nonzero contributions to EW oblique parameters, **dark sector multiplets should couple to the SM Higgs doublet**

1. **Fermionic multiplets**
   - **1 vector-like fermionic $\mathrm{SU(2)_L}$ multiplet:** the Z_2 symmetry for stabilizing DM forbids the multiplet coupling to the Higgs $\Rightarrow$ $S = T = U = 0$
   - **2 types of vector-like $\mathrm{SU(2)_L}$ multiplets whose dimensions differ by one:** Yukawa couplings split the components $\Rightarrow$ Nonzero oblique parameters
2. **Scalar multiplets**
   - **1 real scalar multiplet Φ:** the quartic coupling $\lambda'\Phi^\dagger\Phi H^\dagger H$ can only induce a common mass shift $\Rightarrow$ $S = T = U = 0$
   - **1 complex scalar multiplet Φ:** the quartic coupling $\lambda''\Phi^\dagger\tau^a\Phi H^\dagger\sigma^a H$ can induce mass splittings $\Rightarrow$ Nonzero oblique parameters
   - **≥ 2 scalar multiplets:** various trilinear and quartic couplings could break the mass degeneracy $\Rightarrow$ Nonzero oblique parameters

Direct Detection

For a **Majorana DM candidate** χ, the couplings to the Higgs and Z bosons

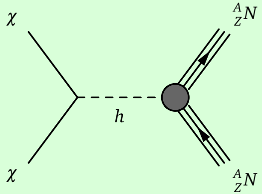
$$\mathcal{L} \supset \frac{1}{2} g_{h\chi\chi} h \bar{\chi}\chi + \frac{1}{2} g_{Z\chi\chi} Z_\mu \bar{\chi}\gamma^\mu\gamma_5\chi$$

would induce **spin-independent (SI)** and **spin-dependent (SD)** DM-nucleus scatterings.

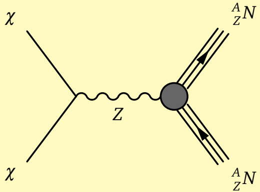
For scalar multiplets, interactions with the Higgs doublet could split the real and imaginary parts of neutral components, leading to a **CP-even or CP-odd real scalar DM candidate**. Its coupling to the Higgs boson would induce **SI scatterings**.

Most stringent constraints from current direct detection experiments:

- **SI:** PandaX-II [1607.07400], LUX [1608.07648]
- **SD:** PICO (proton) [1503.00008, 1510.07754], LUX (neutron) [1602.03489]



SI for both fermionic and scalar χ



SD for Majorana χ
SI for complex scalar χ

Fermionic Models

Introduce 3 Weyl spinors in the dark sector of each model

1. **Singlet-Doublet Fermionic Dark Matter (SDFDM):**

$$S \in (\mathbf{1}, 0), \quad D_1 = \begin{pmatrix} D_1^0 \\ D_1^- \end{pmatrix} \in (\mathbf{2}, -1/2), \quad D_2 = \begin{pmatrix} D_2^+ \\ D_2^0 \end{pmatrix} \in (\mathbf{2}, +1/2)$$

$$\mathcal{L} \supset -\frac{1}{2} m_S SS - m_D \epsilon_{ij} D_1^i D_2^j + y_1 H_i S D_1^i - y_2 H_i^\dagger S D_2^i + \text{h.c.}$$

2. **Doublet-Triplet Fermionic Dark Matter (DTFDM):**

$$D_1 = \begin{pmatrix} D_1^0 \\ D_1^- \end{pmatrix} \in (\mathbf{2}, -1/2), \quad D_2 = \begin{pmatrix} D_2^+ \\ D_2^0 \end{pmatrix} \in (\mathbf{2}, +1/2), \quad T = \begin{pmatrix} T^+ \\ T^0 \\ T^- \end{pmatrix} \in (\mathbf{3}, 0)$$

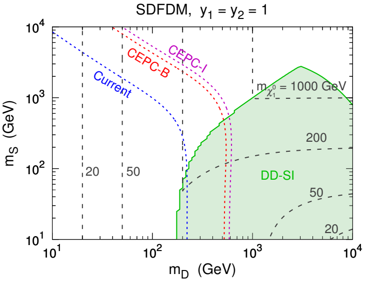
$$\mathcal{L} \supset m_D \epsilon_{ij} D_1^i D_2^j - \frac{1}{2} m_T T^a T^a + y_1 H_i T^a (\sigma^a)^i_j D_1^j - y_2 H_i^\dagger T^a (\sigma^a)^i_j D_2^j + \text{h.c.}$$

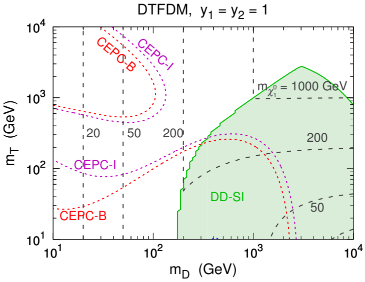
3. **Triplet-Quadruplet Fermionic Dark Matter (TQFDM):**

$$T = \begin{pmatrix} T^+ \\ T^0 \\ T^- \end{pmatrix} \in (\mathbf{3}, 0), \quad Q_1 = \begin{pmatrix} Q_1^+ \\ Q_1^0 \\ Q_1^- \\ Q_1^{--} \end{pmatrix} \in (\mathbf{4}, -1/2), \quad Q_2 = \begin{pmatrix} Q_2^{++} \\ Q_2^+ \\ Q_2^0 \\ Q_2^- \end{pmatrix} \in (\mathbf{4}, +1/2)$$

$$\mathcal{L} \supset -\frac{1}{2} m_T TT - m_Q Q_1 Q_2 + y_1 \epsilon_{jl} (Q_1)_i^{jk} T_k^i H^l - y_2 (Q_2)_i^{jk} T_k^i H_j^\dagger + \text{h.c.}$$

$y_1 = y_2 = 1$ (Custodial Symmetry)

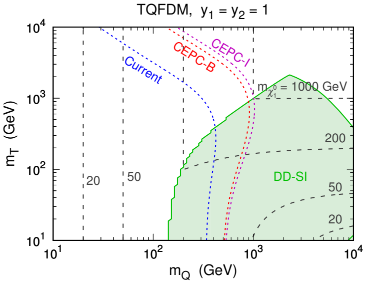




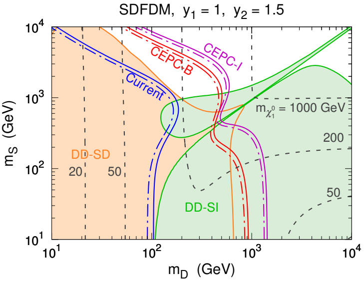
Dotted lines: expected 95% CL constraints from **current**, **CEPC-B**, and **CEPC-I** precisions of EW oblique parameters assuming $T = U = 0$

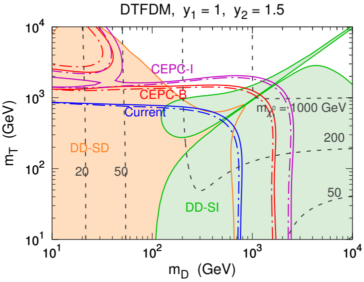
DD-SI: excluded by spin-independent direct detection at 90% CL

Dashed lines: DM particle mass



$y_1 = 1$ and $y_2 = 1.5$ (Custodial Symmetry Violation)

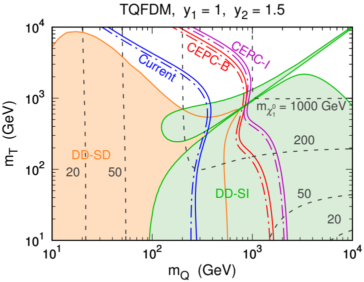




Expected 95% CL constraints from **current**, **CEPC-B**, and **CEPC-I** precisions of EW oblique parameters

Dot-dashed lines: free S, T, and U
Solid lines: assuming $U = 0$

DD-SI: excluded by SI direct detection
DD-SD: excluded by SD direct detection

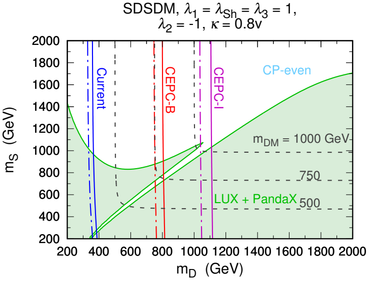


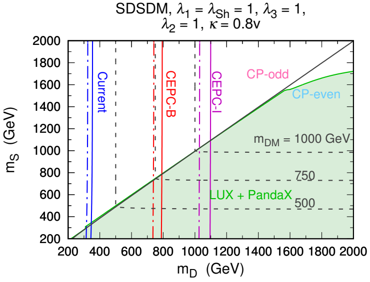
Singlet-Doublet Scalar Dark Matter (SDSDM)

A **real singlet scalar** $S \in (\mathbf{1}, 0)$ and a **complex doublet scalar** $\Phi \in (\mathbf{2}, 1/2)$:

$$\mathcal{L} \supset \frac{1}{2}(\partial_\mu S)^2 - \frac{1}{2} m_S^2 S^2 + (D_\mu \Phi)^\dagger D^\mu \Phi - m_D^2 |\Phi|^2 - (\kappa S \Phi^\dagger H + \text{h.c.}) - \frac{1}{2}\lambda_{Sh} S^2 |H|^2$$
$$-\lambda_1 |H|^2 |\Phi|^2 - [\lambda_2 (\Phi^\dagger H)^2 + \text{h.c.}] - \lambda_3 |\Phi^\dagger H|^2$$

- Custodial symmetry: (a) $\lambda_3 = 2\lambda_2$; b) $\lambda_3 = -2\lambda_2$ and $\kappa = 0$.
- The DM candidate can be either a CP-even or CP-odd scalar.



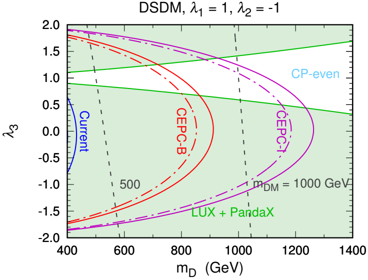


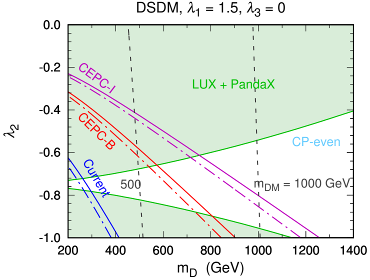
Dot-dashed lines: free S, T, and U **Solid lines:** assuming $U = 0$

Reduction to the Inert Higgs Doublet Model

In the limit $\kappa = 0$ and $m_S \to \infty$, the singlet decouples the SDSDM model reduces to the **inert Higgs doublet model** [Deshpande & Ma, PRD 18, 2574 (1978)]

- $\lambda_2 < 0$: **CP-even** DM candidate, coupling to the Higgs $\propto \lambda_1 + 2\lambda_2 + \lambda_3$
- $\lambda_2 > 0$: **CP-odd** DM candidate, coupling to the Higgs $\propto \lambda_1 - 2\lambda_2 + \lambda_3$
- $\lambda_3 > 2|\lambda_2|$: the DM candidate becomes **unstable** because the charged scalar in the dark sector is lighter





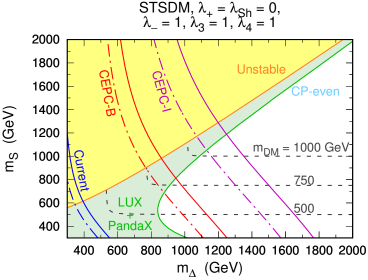
Dot-dashed lines: free S, T, and U **Solid lines:** assuming $U = 0$

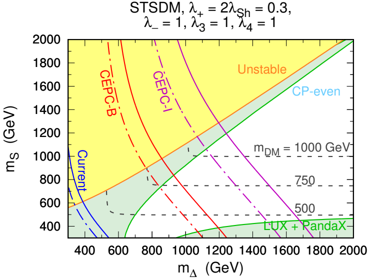
Singlet-Triplet Scalar Dark Matter (STSDM)

A **real singlet scalar** $S \in (\mathbf{1}, 0)$ and a **complex triplet scalar** $\Delta \in (\mathbf{3}, 0)$:

$$-\mathcal{L} \supset \frac{1}{2} m_S^2 S^2 + m_\Delta^2 |\Delta|^2 + \frac{1}{2}\lambda_{Sh} S^2 |H|^2 + \lambda_0 |H|^2 |\Delta|^2 + \lambda_1 H_i^\dagger \Delta_j^i (\Delta^\dagger)_k^j H^k$$
$$+\lambda_2 H_i^\dagger (\Delta^\dagger)_j^i \Delta_k^j H^k - (\lambda_3 H_i^\dagger \Delta_j^i \Delta_k^j H^k + \lambda_3' |H|^2 \Delta_j^i \Delta_i^j + \lambda_4 S H_i^\dagger \Delta_j^i H^j + \text{h.c.})$$

- Define $\lambda_\pm \equiv \lambda_1 \pm \lambda_2$, and λ_3' and λ_0 can be absorbed into λ_3 and λ_+
- Custodial symmetry: $\lambda_- = \lambda_4 = 0$





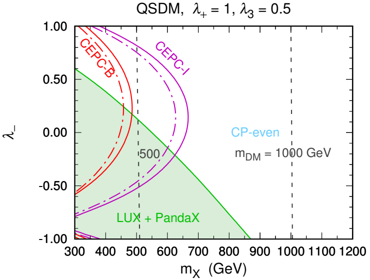
Dot-dashed lines: assuming $S = 0$ **Solid lines:** assuming $S = U = 0$

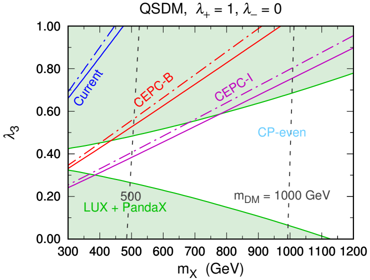
Quadruplet Scalar Dark Matter (QSDM)

A **complex quadruplet scalar** $X \in (\mathbf{4}, 1/2)$:

$$-\mathcal{L} \supset m_X^2|X|^2 + \lambda_0|H|^2|X|^2 + \lambda_1 H_i^\dagger X_k^{ij}(X^\dagger)_{jl}^k H^l + \lambda_2 H_i^\dagger (X^\dagger)_{jk}^i X_l^{jk} H^l$$
$$-(\lambda_3 H_i^\dagger H_j^\dagger X_l^{ik} X_k^{jl} + \text{h.c.})$$

- Define $\lambda_\pm \equiv \lambda_1 \pm \lambda_2$, and λ_0 can be absorbed into λ_+ in the unitary gauge
- Custodial symmetry: $\lambda_- = \pm 2\lambda_3$





Dot-dashed lines: free S, T, and U **Solid lines:** assuming $U = 0$

Conclusions

1. An ultra high precision of **electroweak oblique parameters** in the future **CEPC** project can provide us an excellent opportunity to indirectly probe **weakly interacting dark matter**.
2. We study various fermionic and scalar dark matter models, and show that CEPC can explore these models up to the **TeV scale** and investigate some parameter regions where direct detection cannot reach.

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1. An ultra high precision of **electroweak oblique parameters** in the future **CEPC** project can provide us an excellent opportunity to indirectly probe **weakly interacting dark matter**.
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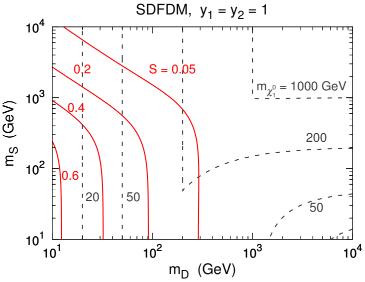
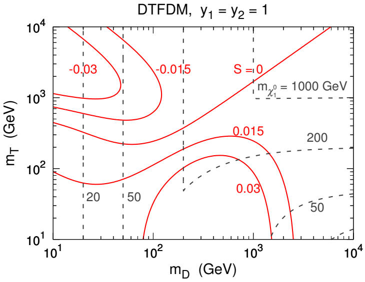
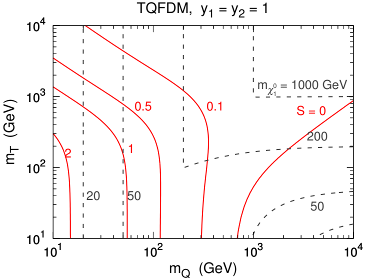
Thanks for your attention!

Contours of S for $y_1 = y_2 = 1$ (Custodial Symmetry)





The behaviors of S in the SDFDM and TQFDM models are similar, while that in the DTFDM model is quite different



SDFDM & TQFDM: one dark sector fermion ($\chi^{\pm}$ or $\chi^{\pm\pm}$) remains unmixed

DTFDM: all dark sector fermion mix with others $\Rightarrow$ cancellation effects for S

Singlet-Doublet Fermionic Dark Matter (SDFDM)

Introduce left-handed Weyl fermions in the dark sector:

$$S \in (\mathbf{1}, 0), \quad D_1 = \begin{pmatrix} D_1^0 \\ D_1^- \end{pmatrix} \in (\mathbf{2}, -1/2), \quad D_2 = \begin{pmatrix} D_2^+ \\ D_2^0 \end{pmatrix} \in (\mathbf{2}, +1/2)$$

$$\mathcal{L}_{\mathrm{S}} = iS^\dagger \bar{\sigma}^\mu \partial_\mu S - \frac{1}{2}(m_S SS + \mathrm{h.c.})$$

$$\mathcal{L}_{\mathrm{D}} = iD_1^\dagger \bar{\sigma}^\mu D_\mu D_1 + iD_2^\dagger \bar{\sigma}^\mu D_\mu D_2 - (m_D \epsilon_{ij} D_1^i D_2^j + \mathrm{h.c.})$$

Yukawa couplings: $\mathcal{L}_{\mathrm{HSD}} = y_1 H_i S D_1^i - y_2 H_i^\dagger S D_2^i + \mathrm{h.c.}$

Custodial symmetry limit $y = y_1 = y_2 \;\Rightarrow\; \mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{SU}(2)_{\mathrm{R}}$ invariant form:

$$\mathcal{L}_{\mathrm{D}} + \mathcal{L}_{\mathrm{HSD}} = i\mathcal{D}_A^\dagger \bar{\sigma}^\mu D_\mu \mathcal{D}^A - \frac{1}{2}[m_D \epsilon_{AB} \epsilon_{ij} (\mathcal{D}^A)^i (\mathcal{D}^B)^j + \mathrm{h.c.}] + [y \epsilon_{AB} (\mathcal{H}^A)_i S (\mathcal{D}^B)^j + \mathrm{h.c.}]$$

$\mathrm{SU}(2)_{\mathrm{R}}$ doublets: $(\mathcal{D}^A)^i = \begin{pmatrix} D_1^i \\ D_2^i \end{pmatrix}, \quad (\mathcal{H}^A)_i = \begin{pmatrix} H_i^\dagger \\ H_i \end{pmatrix}$

SDFDM: State Mixing

The dark sector involves 3 Majorana fermions and 1 singly charged fermion

$$\mathcal{L}_{\text{mass}} = -\frac{1}{2}(S \quad D_1^0 \quad D_2^0)\mathcal{M}_{\text{N}} \begin{pmatrix} S \\ D_1^0 \\ D_2^0 \end{pmatrix} - m_D D_1^- D_2^+ + \text{h.c.} = -\frac{1}{2}\sum_{i=1}^{3} m_{\chi_i^0}\chi_i^0\chi_i^0 - m_{\chi^\pm}\chi^-\chi^+ + \text{h.c.}$$

$$\mathcal{M}_{\text{N}} = \begin{pmatrix} m_S & \frac{1}{\sqrt{2}}y_1 v & \frac{1}{\sqrt{2}}y_2 v \\ \frac{1}{\sqrt{2}}y_1 v & 0 & -m_D \\ \frac{1}{\sqrt{2}}y_2 v & -m_D & 0 \end{pmatrix}, \quad \begin{pmatrix} S \\ D_1^0 \\ D_2^0 \end{pmatrix} = \mathcal{N}\begin{pmatrix} \chi_1^0 \\ \chi_2^0 \\ \chi_3^0 \end{pmatrix}$$

$$\mathcal{N}^{\text{T}}\mathcal{M}_{\text{N}}\mathcal{N} = \text{diag}(m_{\chi_1^0}, m_{\chi_2^0}, m_{\chi_3^0}), \quad \chi^+ = D_2^+, \quad \chi^- = D_1^-$$

Couplings of the **DM candidate** χ_1^0 to the Higgs and Z bosons:

$$\mathcal{L} \supset \frac{1}{2}g_{h\chi_1^0\chi_1^0} h\bar{\chi}_1^0\chi_1^0 + \frac{1}{2}g_{Z\chi_1^0\chi_1^0} Z_\mu \bar{\chi}_1^0\gamma^\mu\gamma_5\chi_1^0$$

$$g_{h\chi_1^0\chi_1^0} = -\sqrt{2}(y_1\mathcal{N}_{21} + y_2\mathcal{N}_{31})\mathcal{N}_{11}, \quad g_{Z\chi_1^0\chi_1^0} = -\frac{g}{2c_{\text{W}}}(|\mathcal{N}_{21}|^2 - |\mathcal{N}_{31}|^2)$$

Custodial symmetry limit $y_1 = y_2 \;\Rightarrow\; T = U = 0$ and $g_{Z\chi_1^0\chi_1^0} = 0$

$y_1 = y_2$ and $m_D < m_S \;\Rightarrow\; g_{h\chi_1^0\chi_1^0} = 0$

Doublet-Triplet Fermionic Dark Matter (DTFDM)

Introduce left-handed Weyl fermions in the dark sector:

$$D_1 = \begin{pmatrix} D_1^0 \\ D_1^- \end{pmatrix} \in (\mathbf{2}, -1/2), \quad D_2 = \begin{pmatrix} D_2^+ \\ D_2^0 \end{pmatrix} \in (\mathbf{2}, +1/2), \quad T = \begin{pmatrix} T^+ \\ T^0 \\ T^- \end{pmatrix} \in (\mathbf{3}, 0)$$

$$\mathcal{L}_\mathrm{D} = iD_1^\dagger \bar{\sigma}^\mu D_\mu D_1 + iD_2^\dagger \bar{\sigma}^\mu D_\mu D_2 + (m_D \epsilon_{ij} D_1^i D_2^j + \mathrm{h.c.})$$

$$\mathcal{L}_\mathrm{T} = iT^\dagger \bar{\sigma}^\mu D_\mu T - \frac{1}{2}(m_T T^a T^a + \mathrm{h.c.})$$

Yukawa couplings: $\mathcal{L}_\mathrm{HDT} = y_1 H_i T^a (\sigma^a)^i_j D_1^j - y_2 H_i^\dagger T^a (\sigma^a)^i_j D_2^j + \mathrm{h.c.}$

Custodial symmetry limit $y = y_1 = y_2$ $\Rightarrow$ $\mathrm{SU(2)_L \times SU(2)_R}$ invariant form:

$$\mathcal{L}_\mathrm{D} + \mathcal{L}_\mathrm{HDT} = i\mathcal{D}_A^\dagger \bar{\sigma}^\mu D_\mu \mathcal{D}^A + \frac{1}{2}[m_D \epsilon_{AB} \epsilon_{ij} (\mathcal{D}^A)^i (\mathcal{D}^B)^j + \mathrm{h.c.}] + [y \epsilon_{AB} (\mathcal{H}^A)_i T^a (\sigma^a)^i_j (\mathcal{D}^B)^j + \mathrm{h.c.}]$$

$\mathrm{SU(2)_R}$ doublets: $(\mathcal{D}^A)^i = \begin{pmatrix} D_1^i \\ D_2^i \end{pmatrix}$, $(\mathcal{H}^A)_i = \begin{pmatrix} H_i^\dagger \\ H_i \end{pmatrix}$

DTFDM: State Mixing

The dark sector involves 3 Majorana fermions and 2 singly charged fermions

$$\mathcal{L}_{\text{mass}} = -\frac{1}{2}(T^0 \quad D_1^0 \quad D_2^0)\mathcal{M}_{\text{N}} \begin{pmatrix} T^0 \\ D_1^0 \\ D_2^0 \end{pmatrix} - (T^- \quad D_1^-)\mathcal{M}_{\text{C}} \begin{pmatrix} T^+ \\ D_2^+ \end{pmatrix} + \text{h.c.}$$

$$= -\frac{1}{2}\sum_{i=1}^{3} m_{\chi_i^0} \chi_i^0 \chi_i^0 - \sum_{i=1}^{2} m_{\chi_i^\pm} \chi_i^- \chi_i^+ + \text{h.c.}$$

$$\mathcal{M}_{\text{N}} = \begin{pmatrix} m_T & \frac{1}{\sqrt{2}} y_1 v & -\frac{1}{\sqrt{2}} y_2 v \\ \frac{1}{\sqrt{2}} y_1 v & 0 & m_D \\ -\frac{1}{\sqrt{2}} y_2 v & m_D & 0 \end{pmatrix}, \quad \mathcal{M}_{\text{C}} = \begin{pmatrix} m_T & -y_2 v \\ -y_1 v & -m_D \end{pmatrix}$$

$$\begin{pmatrix} T^0 \\ D_1^0 \\ D_2^0 \end{pmatrix} = \mathcal{N} \begin{pmatrix} \chi_1^0 \\ \chi_2^0 \\ \chi_3^0 \end{pmatrix}, \quad \begin{pmatrix} T^+ \\ D_2^+ \end{pmatrix} = \mathcal{C}_{\text{L}} \begin{pmatrix} \chi_1^+ \\ \chi_2^+ \end{pmatrix}, \quad \begin{pmatrix} T^- \\ D_1^- \end{pmatrix} = \mathcal{C}_{\text{R}} \begin{pmatrix} \chi_1^- \\ \chi_2^- \end{pmatrix}$$

Custodial symmetry limit $y_1 = y_2 \Rightarrow T = U = 0$ and $g_{Z\chi_1^0\chi_1^0} = 0$

$y_1 = y_2$ and $m_D < m_T \Rightarrow g_{h\chi_1^0\chi_1^0} = 0$

Triplet-Quadruplet Fermionic Dark Matter (TQFDM)

Introduce left-handed Weyl fermions in the dark sector:

$$T = \begin{pmatrix} T^+ \\ T^0 \\ T^- \end{pmatrix} \in (\mathbf{3}, 0), \quad Q_1 = \begin{pmatrix} Q_1^+ \\ Q_1^0 \\ Q_1^- \\ Q_1^{--} \end{pmatrix} \in (\mathbf{4}, -1/2), \quad Q_2 = \begin{pmatrix} Q_2^{++} \\ Q_2^+ \\ Q_2^0 \\ Q_2^- \end{pmatrix} \in (\mathbf{4}, +1/2)$$

$$\mathcal{L}_\mathrm{T} = iT^\dagger \bar{\sigma}^\mu D_\mu T - \frac{1}{2}(m_T T T + \mathrm{h.c.})$$

$$\mathcal{L}_\mathrm{Q} = iQ_1^\dagger \bar{\sigma}^\mu D_\mu Q_1 + iQ_2^\dagger \bar{\sigma}^\mu D_\mu Q_2 - (m_Q Q_1 Q_2 + \mathrm{h.c.})$$

Yukawa couplings: $\mathcal{L}_\mathrm{HTQ} = y_1 \epsilon_{jl} (Q_1)_i^{jk} T_k^i H^l - y_2 (Q_2)_i^{jk} T_k^i H_j^\dagger + \mathrm{h.c.}$

Custodial symmetry limit $y = y_1 = y_2 \;\Rightarrow\; \mathrm{SU}(2)_\mathrm{L} \times \mathrm{SU}(2)_\mathrm{R}$ invariant form:

$$\mathcal{L}_\mathrm{Q} + \mathcal{L}_\mathrm{HTQ} = i\mathcal{Q}_A^\dagger \bar{\sigma}^\mu D_\mu \mathcal{Q}^A - \frac{1}{2}[m_Q \epsilon_{AB} \epsilon_{il} (\mathcal{Q}^A)_k^{ij} (\mathcal{Q}^B)_j^{lk} + \mathrm{h.c.}] + [y \epsilon_{AB} (\mathcal{Q}^A)_i^{jk} T_k^i (\mathcal{H}^B)_j + \mathrm{h.c.}]$$

$$\mathrm{SU}(2)_\mathrm{R} \text{ doublets: } (\mathcal{Q}^A)_k^{ij} = \begin{pmatrix} (Q_1)_k^{ij} \\ (Q_2)_k^{ij} \end{pmatrix}, \quad (\mathcal{H}^A)_i = \begin{pmatrix} H_i^\dagger \\ H_i \end{pmatrix}$$

TQFDM: State Mixing

3 Majorana fermions, 3 singly charged fermions, 1 doubly charged fermion

$$\begin{aligned}\mathcal{L}_{\text{mass}} &= -\frac{1}{2}(T^0, Q_1^0, Q_2^0)\mathcal{M}_{\text{N}}\begin{pmatrix}T^0\\Q_1^0\\Q_2^0\end{pmatrix} - (T^-, Q_1^-, Q_2^-)\mathcal{M}_{\text{C}}\begin{pmatrix}T^+\\Q_1^+\\Q_2^+\end{pmatrix} - m_Q Q_1^{--}Q_2^{++} + \text{h.c.}\\ &= -\frac{1}{2}\sum_{i=1}^{3} m_{\chi_i^0}\chi_i^0\chi_i^0 - \sum_{i=1}^{3} m_{\chi_i^\pm}\chi_i^-\chi_i^+ - m_Q\chi^{--}\chi^{++} + \text{h.c.}\end{aligned}$$

$$\mathcal{M}_{\text{N}} = \begin{pmatrix} m_T & \frac{1}{\sqrt{3}}y_1 v & -\frac{1}{\sqrt{3}}y_2 v\\ \frac{1}{\sqrt{3}}y_1 v & 0 & m_Q\\ -\frac{1}{\sqrt{3}}y_2 v & m_Q & 0\end{pmatrix}, \quad \mathcal{M}_{\text{C}} = \begin{pmatrix} m_T & \frac{1}{\sqrt{2}}y_1 v & -\frac{1}{\sqrt{6}}y_2 v\\ -\frac{1}{\sqrt{6}}y_1 v & 0 & -m_Q\\ \frac{1}{\sqrt{2}}y_2 v & -m_Q & 0\end{pmatrix}$$

$$\begin{pmatrix}T^0\\Q_1^0\\Q_2^0\end{pmatrix} = \mathcal{N}\begin{pmatrix}\chi_1^0\\\chi_2^0\\\chi_3^0\end{pmatrix}, \quad \begin{pmatrix}T^+\\Q_1^+\\Q_2^+\end{pmatrix} = \mathcal{C}_L\begin{pmatrix}\chi_1^+\\\chi_2^+\\\chi_3^+\end{pmatrix}, \quad \begin{pmatrix}T^-\\Q_1^-\\Q_2^-\end{pmatrix} = \mathcal{C}_R\begin{pmatrix}\chi_1^-\\\chi_2^-\\\chi_3^-\end{pmatrix}, \quad \begin{aligned}\chi^{--} &\equiv Q_1^{--}\\ \chi^{++} &\equiv Q_2^{++}\end{aligned}$$

Custodial symmetry limit $y_1 = y_2 \;\Rightarrow\; T = U = 0$ and $g_{Z\chi_1^0\chi_1^0} = 0$

$y_1 = y_2$ and $m_Q < m_T \;\Rightarrow\; g_{h\chi_1^0\chi_1^0} = 0$